Assignment 1

1.2 A uniform spool of mass M and diameter d rests on end on a frictionless table. A massless string wrapped around the spool is attached to a weight m which hangs over the edge of the table. If the spool is released from rest when its center of mass is a distance ℓ from the edge of the table, what is the velocity of the weight m when the center of mass of the spool reaches the edge of the table?

1.4 A rocket with initial mass m_0 emits exhaust gases at a constant rate m_0/τ with constant speed v_0 relative to the rocket. It starts from rest at the earth's surface and rises vertically. Treating the earth's gravitational field as uniform, find the height h as a function of time. Discuss the behavior for $t \ll \tau$ and for $t \to \tau$. Sketch carefully the resulting speed and displacement.

1.7 A rocket of mass m is in circular orbit around the earth at a distance R from the center.
(a) What tangential impulse, that is, $m\Delta v$, must be given to the body so that it just escapes to infinity?
(b) Describe the resulting orbit.
(c) Compare with the radial impulse that must be given to a particle initially at rest at R if it is to acquire sufficient velocity to escape to infinity.

1.11 A particle of mass m moves in a singular central potential with $V(r) = -\lambda r^{-n}$ with $n > 2$. Reduce the equations of motion to an equivalent one-dimensional problem and discuss the qualitative nature of the orbit for different values of the energy. For the bound orbits, show that the particle takes a *finite* length of time to spiral into the center of force, passing through a *finite* number of revolutions. Can you say anything about the subsequent motion?

Packet #1 An Earth satellite of mass m is placed in a circular orbit. Due to the fact that space is not an ideal vacuum, the satellite is subject to an extra frictional force $\mathbf{F}$, which we assume is linear in the satellite velocity $\mathbf{v}$, i.e. $\mathbf{F} = -A\mathbf{v}$ where A is a constant. This force dissipates the satellite energy so that eventually the spacecraft hits the ground, which determines its lifetime (in reality the drag constant A is a function of altitude and satellites often burn in upper layers of atmosphere, but we will ignore this). Assuming that the energy dissipated by friction during one full revolution is much smaller than the total energy, compute the lifetime of the satellite. Assume the Earth can be modeled by a sphere, the initial radius of the orbit is 10 times as big as the Earth radius, and the satellite is much lighter than the Earth.