Assignment 10 Hints

5.3 – In (a), get the equation of motion using ordinary force-diagram analysis. The reaction forces are the normal force and the friction force. Determine them from the acceleration of the center of mass, and then use the corresponding torques to get $d\mathbf{L}_{cm}/dt$. Note that this is the derivative in the intertial frame, and here $\mathbf{L}_{cm}$ is the angular momentum of the body with respect to the center-of-mass, not the angular momentum *of* the center of mass.

In (b), you can pretty much work out $\mathbf{\Omega}$ from the geometry directly. For instance, θ is the rotation angle about the $\hat{\mathbf{e}}_1$ axis, so $\Omega_1 = \theta$. The other two are just a bit harder. However, if you want a more systematic approach here is how you can get it. Start by writing out the $\hat{\mathbf{e}}_s$ vectors in terms of the inertial coordinates. Take the derivatives with respect to time to get formulas for $d\hat{\mathbf{e}}_s/dt$. But these must be equal to $\mathbf{\Omega} \times \hat{\mathbf{e}}_s$. One way to solve for $\mathbf{\Omega}$ is to write out the cross product as a matrix multiplication,

$$\frac{d}{dt}\begin{bmatrix} e_{s1} \\ e_{s2} \\ e_{s3} \end{bmatrix} = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \begin{bmatrix} e_{s1} \\ e_{s2} \\ e_{s3} \end{bmatrix},$$

from which it is easy to read off the Ω components. However, these are the components in the inertial frame. To get $\mathbf{\Omega}$ in the body coordinates, just use $\Omega_s = \hat{\mathbf{e}}_s \cdot \mathbf{\Omega}$. I found it pretty instructive to go through this method.

Once you have Ω, the rest is easy. The relation the problem refers to in part (c) is

$$\left(\frac{d\mathbf{L}}{dt}\right)_{\text{inert}} = \left(\frac{d\mathbf{L}}{dt}\right)_{\text{body}} + \mathbf{\Omega} \times \mathbf{L}.$$

You can get the body derivative term from the expression in (b).

5.9 – For part (a), the brute force approach seems like it gets pretty ugly. It works better to first use $p_\alpha = p_\gamma$ to simplify V_{eff}, and then use trig relations to simplify it further. I reduced it to a sum of two simple terms (plus a constant) before doing any expansion.

In part (b), feel free to use a computer to plot V_{eff}. The tricky part here is understanding why (31.16) fails. Just follow the steps of the derivation, putting in $\beta = 0$ as you go. That should make the problem pretty obvious.

5.10 – In part (b), the equation for β referred to is Eq. (31.6b), not (31.5b). You'll need to get the energy from the initial conditions. I was able to go from there to the equation for u easily enough. To get the solution, I took the square root, substituted $v = u - 1$, and then looked up the resulting integral. If you follow this approach, you'll need the half-angle identity for the hyperbolic tangent,

$$\tanh^2 \frac{x}{2} = \frac{\cosh x - 1}{\cosh x + 1},$$

to get the solution to the listed form.

Packet #8 – You'll notice that ψ, ϕ, and θ are not defined in the problem. That's bad form on the part of whoever wrote the question, and I would hope to avoid that myself. But I can't guarantee that you won't get a badly worded problem on the actual exam... be prepared to make a reasonable guess about what a question is supposed to mean.