

Assignment 8 Hints

5.2 – Make sure you are clear on the picture here. The body is spinning rapidly about its symmetry axis, and is held in some kind of cage that confines the symmetry axis to a horizontal plane, but allows the axis to rotate freely within that plane.

Part (a) is pretty straightforward. Just keep in mind that we are dealing with two separate rotating frames, the earth frame and the body frame. The Coriolis force on mass m_p depends on its velocity in the Earth frame. You can relate the velocity in the earth frame to the rotation rate of the body frame (relative to the earth) using the normal frame transformation rule. Note that the answer is expressed as a sum over discrete mass elements, but it could just as well be expressed as an integral over the density.

For part (b), set up a coordinate frame with \hat{e}_1 vertical (relative to the Earth), \hat{e}_3 fixed to the symmetry axis of the top, and $\hat{e}_2 = \hat{e}_3 \times \hat{e}_1$. Define the angle between the \hat{e}_3 axis and north to be ϕ . Then $\dot{\phi}$ is the 1 component of the body's angular velocity $\mathbf{\Omega}$ (since it describes rotation around the 1 axis). You should be able to show that ϕ obeys the equation of motion $I_1 \ddot{\phi} = \Gamma_1$.

Then you need to work out the torque $\mathbf{\Gamma}$ in terms of these coordinates. (You'll need an expression for ω in these coordinates too.) The sum in the torque expression eventually reduces to various components of the inertia tensor, though you have to use symmetry to note that

$$\sum_p m_p r_{p1}^2 = \sum_p m_p r_{p2}^2,$$

as well as

$$\sum_p m_p r_{p1} r_{p2} = 0$$

Also, you can approximate $\Omega = (\Omega_1^2 + \Omega_3^2)^{1/2}$ as Ω_3 , since expect $\Omega_3 \gg \Omega_1$.

5.5 – For part (a), it is fine to simply take the derivatives of E and L^2 and show that they are zero according to Euler's equations.

For part (b), I solved the E and L^2 equations for ω_1 and ω_2 , and substituted into the Euler equation for ω_3 . It is messy though; I get

$$t - t_0 = \int_{\omega_3(0)}^{\omega_3(t)} \frac{I_3 \sqrt{I_1 I_2}}{\sqrt{-A + B\omega_3^2 - C\omega_3^4}} d\omega_3$$

with $A = (L^2 - 2EI_1)(L^2 - 2EI_2)$,

$$B = I_3[(2I_3 - I_1 - I_2)L^2 + 2E(2I_1I_2 - I_1I_3 - I_2I_3)],$$

$$C = I_3^2(I_3 - I_2)(I_3 - I_1).$$

For the formal solution to the whole vector, just make sure you have expressions for ω_1 and ω_2 in terms of ω_3 .

5.6 – This is similar to problem 5.5, but here solve for ω_1 and ω_3 in terms of ω_2 , and plug into the equation for $d\omega_2/dt$. You can either rearrange indices in the result from 5.5, or you can rederive it, taking advantage from the outset that the expressions simplify when you use the relation between E and L^2 and express both in terms of ω_∞ . This simplification lets you do the integral.

Note that if you run this evolution backwards in time, it gives an example of what can happen when you try to rotate a body about the its intermediate principle axis.