Assignment 6 Hints

2.1 – The idea here is to put the Lorentz force into the non-inertial equation of motion (10.2), and show that the magnetic force and the Coriolis force cancel. This isn't too hard, but make sure you properly transform the $\mathbf{r}$ and $\mathbf{v}$ in the Lorentz law to the corresponding quantities in the rotating frame. (It turns out that nothing changes, but you need to derive that fact.) For this to work the centrifugal term has to be negligble, which is how you get a limit on how large $\mathbf{B}$ can be.

There's not much to part (b), just show that if it works for one particle then it works for all of them, and that the rotation doesn't affect the interaction terms.

2.2 – We talked about this a little bit in class. The idea is that for a coordinate frame on the surface of the earth, the centrifugal force is much reduced, since the position vector $\mathbf{r}$ is now much smaller. But to compensate, there is now a $d^2\mathbf{R}/dt^2$ term, since the coordinate origin is accelerating. You need to show that the sum of the remaining centrifugal term and the acceleration term give the same effect as the centrifugal term does when the origin is at the center of the earth. You can assume that the particle is at the origin of the surface coordinates (or close enough that the difference is negligible).

2.5 – Do this using the method I discussed in class: first solve for the trajectory $\mathbf{r}_0$ ignoring the Coriolis force, and then use $\mathbf{r}_0$ to calculate the first order correction, $\mathbf{r}_1$. In part (a), you only need the x component of $\mathbf{r_1}$, but for (b) you need all three. Furthermore, to get the new range you need to calculate the new time when the ball lands, that is, solve $z(t_1) = z_0(t_1) + z_1(t_1) = 0$ for t_1. Then the new range is $y(t_1)$ as compared to the unmodified $y_0(t_0)$. You will need to Taylor expand $y(t_1)$ to first order in ω (not forgetting that t_1 itself depends on ω). Once that's done, you need to express t_0 and V_0 in terms of R, and you should have the quoted result.

2.7 – For part (a), note that for any function ϕ, the vector $\nabla\phi$ is normal to the surface $\phi = \text{const}$. You can prove that by recalling $\hat{\mathbf{n}} \cdot \nabla\phi$ gives the derivative in ϕ along the direction $\hat{\mathbf{n}}$.

For parts (b) and (c), I think the textbook makes it a little harder than necessary, so I modified the question a bit. Note that J_2 is a dimensionless constant, not the second order Bessel function.

For part (b), recall that the geoid is the surface of constant Φ. So if R_e is the radius at the equator, then we know $\Phi(R, \theta) = \Phi(R_e, \theta = \pi/2)$. You need to solve that equation for $R(\theta)$. It is easiest to do this by defining $R(\theta) = R_e(1 + \delta)$ and solve for δ. It is also handy to introduce the dimensionless constant

$$K = \frac{\omega^2 R_e^3}{MG} \approx 3.5 \times 10^{-3}.$$

The key is to realize that all three of J_2, K, and δ are small. So any time you have an expression involving any of them, you can Taylor expand it to first order. Do that at the beginning, not the end. Also, terms like $J_2\delta$ are second order, and can be dropped. Once again, eliminate them as soon as possible.

Part (c) works much the same way. You have to take the derivative of the complete expression for Φ, but then immediately Taylor expand everything and drop all terms above first order. At some point, you need to eliminate J_2 in terms of the expression for ΔR from part (b).