Assignment 4 Hints

This problem set has only three problems from the book. That doesn't mean you should budget less time than usual to complete it: problems 4.3 and 4.9 are rather long.

4.3 – In part (c), you don't need to normalize the eigenvectors to describe the corresponding motion. You will need to normalize them for part (d). The algebra there is not trivial. I found useful the relation

$$\frac{m_2}{m_1} = \frac{\gamma^2}{1-\gamma^2}$$

which you should be able to derive without difficulty.

4.4 – For part (a), the idea is to construct the first integral by multiplying the equation of motion by $\dot{\theta}$. For part (b), recall that motion is stable if $\omega^2 > 0$, unstable if $\omega^2 < 0$. Remember you don't need to do the part about the rotating frame, since we haven't covered that yet.

4.9 – You can set this up in various ways. I just used Cartesian coordinates for each mass. To get the potential energy, note that a spring between particles 1 and 2 has potential energy

$$V = \frac{1}{2}k\left(|\mathbf{r_1} - \mathbf{r_2}| - a\right)^2$$

where k is the spring constant and a is the equilibrium length. You will need to expand this for small displacements about equilibrium.

I set it up with particle 3 at the origin, particle 1 at $(a, 0)$ and particle 2 at $(0, a)$. I get a potential matrix

$$\mathsf{V} = \frac{1}{2}k \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -1 & 0 \\ \frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & -1 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where the coordinates are ordered $(x_1, y_2, x_2, y_1, x_3, y_3)$.

You should be able to figure out what the eigenvectors with zero frequency are using physical reasoning. With them in hand, I was able to find the other three eigenvectors by inspection. (The symmetry of the problem gives some clues as to their form, and they must be orthogonal to each other and the first three eigenvectors.) If that doesn't work, you can evaluate the determinant with Mathematica or the like. I wouldn't advise evaluating the determinant by hand.