Assignment 3 Hints

3.5 – There is only one coordinate in this problem, Z. The other relevant parameter is the the height of the bananas from the floor, which we might call Y. Prove that $Y = d - Z + \text{const}$. With that relation, calculation of the Lagrangian is straightforward.

When solving the equation of motion, note that you can integrate $\ddot{d}(t)$ twice to get $d(t)$, even though the functional form of d is not specified. For the final question, you want to show that $Y - Z$ is constant.

3.14 – The examples on pages 71-72 in the text and my lecture 6 (Sept 11) are helpful here.

3.17 – Both (a) and (b) are solved similarly to the example on pages 74-77 of the text. Do note however that only one Lagrange multiplier is needed since only the normal force is of interest here. You may recall that the moment of inertia of a sphere is $\frac{2}{5}MR^2$.

3.19 – The point of this problem is that even on a curved surface, free particle motion always consists of taking the shortest path between two points. (Thus in free space, for instance, particles travel in straight lines.)

In part (a), note that here $L = T = (m/2)(d\mathbf{s}/dt)^2$. In deriving the equations of motion, the sums can be confusing. If necessary, just write them out. (For example, write $q^1 + q^2$ instead of $\sum q^i$.) Note that the inverse tensor is defined by

$$\sum_j g^{ij} g_{jk} = \delta_{ik}.$$

You are supposed to do part (b) independently of (a). You need to minimize the integral

$$I = \int ds = \int \phi d\tau$$

for

$$\phi = \frac{ds}{d\tau} = \left[\sum_{ij} g_{ij} \frac{dq^i}{d\tau} \frac{dq^j}{d\tau} \right]^{1/2}.$$

For a uniform parametization, $ds/d\tau$ is constant, so $d\phi/d\tau = 0$. You need to use this to eliminate some terms in the Euler equations.

In part (c), the idea is to show that t/T is itself a uniform parametrization of the curve satisfying the conditions of (b), where T is the total time duration of the trajectory. Once you have $v = \text{const}$, then it is clear that ds/dt is constant.