Assignment 12 Hints

7.5 – Note that I changed the last part of this problem a little bit. I don't think it should give you much trouble.

7.6 – I used $\rho(x) = 1 - x^2/a^2$ for (a), $\rho(x) = 1 - (x/a)^b$ for (b) (with variational parameter b), and $\rho(x) = (x/a)(1 - x^2/a^2)$ for (c). I got answers of $3.64\tau/a^2\sigma_0$, $3.50\tau/a^2\sigma_0$, and $23.2\tau/a^2\sigma_0$ respectively. You are welcome to use my forms, or try one of your own. However, you will use your form for (a) again in problem 7.10; if you make it too complicated, you wont be able to do the integral there.

7.10 – Parts (a) and (b) are pretty simple. You can show $\delta\omega^2 = 0$ in (b) starting with a similar trick as in Eq. (41.2).

Part (c) is complicated. First you will need to calculate $G_0(x, x')$. This sound intimidating, but you'll see that Eq. (43.13) is quite simple; if you use the solutions to that in 43.29, you should get

$$G_0(x, x') = \frac{a}{2\tau}\left(1 + \frac{x_<}{a}\right)\left(1 - \frac{x_>}{a}\right)$$

(Notice that this is simply the equilibirium shape taken by the string in response to a static force applied at point x'. If that doesn't seem apparent, make a sketch for a few different x's.)

Once you have that, you need to evaluate the integrals in ω^2. You already did the numerator in 7.9. The denominator is some work though, because you need to break it up in to different regions depending on whether $x > 0$, $x' > 0$, and $x > x'$. However, the integrand is symmetric under $x \leftrightarrow x'$, $x \leftrightarrow -x$, and $x' \leftrightarrow -x'$. Using these symmetries, show that there are only two independent integrals:

$$\int_{-a}^{0} dx \int_{-a}^{x} dx' \rho(x)\sigma(x)G_0(x, x')\rho(x')\sigma(x')$$

and

$$\int_{-a}^{0} dx \int_{0}^{a} dx' \rho(x)\sigma(x)G_0(x, x')\rho(x')\sigma(x').$$

You'll have to evaluate these. Using my trial ρ from 7.9(a), it was straightforward but a bit messy; the first integral required multiplying out the terms of a 9th order polynomial. I got an answer of $\omega^2 = 3.487\tau/a^2\sigma_0$.