Assignment 10 Hints

6.4 – The relativistic generalization of Newton's Law is

$$\frac{d}{dt}(\gamma m\mathbf{v}) = -\nabla V(\mathbf{r})$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$. In part (c), the radial coordinats r and ϕ are defined in the usual way.

6.5 – In part (c), you have some flexibility in choosing $\mathbf{A}$; you should take it to have the form $\hat{\mathbf{z}}A_z(x, y)$ as for the previous parts of the problem.

6.8 – I went through 6.7 in class on 11/6, so you shouldn't have too much trouble here. For the rotation part, set up the initial coordinates q_σ as spherical (or cylindrical) coordinates with axis along $\hat{\mathbf{n}}$. Then determine how $\hat{\mathbf{n}} \cdot \mathbf{L}$ is related to the p_σ.

6.12 – Use Cartesian coordinates with $\mathbf{g} = -g\hat{\mathbf{z}}$, and get the Hamiltonian. Note that x and y are cyclic, so you can take

$$S = W(z) + \alpha_x x + \alpha_y y - \alpha_1 t$$

with $\alpha_1 = E$. Put this in the Hamilton-Jacobi equation and work out an integral expression for $W(z)$. (You can do the integral if you want, or take the derivatives first.)

Then set $\beta_\sigma = \partial S/\partial\alpha_\sigma$ to be constant, giving three implicit equations of motion. One relates z and t, and the other two relate x and y to z. The "orbit" here is just the relations $x(z)$ and $y(z)$. Get these relations explicitly, as well as the full solution $\mathbf{r}(t)$. You will also need to determine the α's and β's in terms of the initial position and velocity. Obviously, you know what the answers should be.