Problem C1. In class on 2/8, we considered a two-level system with Hamiltonian $H = H^{(0)} + H'$. Suppose

$$H^{(0)} = \begin{bmatrix} E_1^{(0)} & 0 \\ 0 & E_2^{(0)} \end{bmatrix} \quad \text{and} \quad H' = \begin{bmatrix} H'_{11} & H'_{12} \\ H'_{21} & H'_{22} \end{bmatrix}.$$ 

The eigenstates of this Hamiltonian can be written

$$|\psi_1\rangle = c_{11} |\psi^{(0)}_1\rangle + c_{12} |\psi^{(0)}_2\rangle$$

$$|\psi_2\rangle = c_{21} |\psi^{(0)}_1\rangle + c_{22} |\psi^{(0)}_2\rangle$$

where the $|\psi^{(0)}_n\rangle$ are the eigenstates of $H^{(0)}$. I derived in class that

$$c_{11} = \frac{1}{\sqrt{2}} \left(1 + \frac{\Delta}{\sqrt{\Delta^2 + V^2}}\right)^{1/2} \quad \text{and} \quad c_{21} = \frac{1}{\sqrt{2}} \left(1 - \frac{\Delta}{\sqrt{\Delta^2 + V^2}}\right)^{1/2}$$

for $\Delta = H'_{22} - H'_{11}$ assumed greater than zero, and $H'_{12} = \frac{1}{2} V \exp(i\phi)$. In the course of that solution, we also obtained

$$c_{12} = c_{11} \exp(-i\phi) \left(\frac{\Delta - \sqrt{\Delta^2 + V^2}}{V}\right) \quad \text{and} \quad c_{22} = c_{21} \exp(-i\phi) \left(\frac{\Delta + \sqrt{\Delta^2 + V^2}}{V}\right)$$

(a) Complete the derivation to obtain the final formulas for $c_{12}$ and $c_{22}$,

$$c_{12} = -e^{-i\phi} \frac{1}{\sqrt{2}} \left(1 - \frac{\Delta}{\sqrt{\Delta^2 + V^2}}\right)^{1/2} \quad \text{and} \quad c_{22} = e^{-i\phi} \frac{1}{\sqrt{2}} \left(1 + \frac{\Delta}{\sqrt{\Delta^2 + V^2}}\right)^{1/2}$$

for $\phi = \arg(H'_{12})$.

(b) For the case $\Delta \gg V$, Taylor expand each of the $c_{nm}$ to first order in the elements of $H'$, and compare the correction terms to the general formula (6.12) in the text. Note my notation is slightly different from Griffiths’, with my $c_{nm}$ corresponding to his $c^{(n)}_m$. Also, you’ll need to adjust the arbitrary overall phase of the $|\psi_2\rangle$ state to correct for the fact that $|\psi_2\rangle$ should really be identical to $|\psi^{(0)}_2\rangle$ when $H'$ is vanishingly small.

(c) Determine the $c_{nm}$ for the case $\Delta = 0$. 
