

Problem C1. In class on 2/8, we considered a two-level system with Hamiltonian $H = H^{(0)} + H'$. Suppose

$$H^{(0)} = \begin{bmatrix} E_1^{(0)} & 0 \\ 0 & E_2^{(0)} \end{bmatrix} \quad \text{and} \quad H' = \begin{bmatrix} H'_{11} & H'_{12} \\ H'_{21} & H'_{22} \end{bmatrix}.$$

The eigenstates of this Hamiltonian can be written

$$\begin{aligned} |\psi_1\rangle &= c_{11} |\psi_1^{(0)}\rangle + c_{12} |\psi_2^{(0)}\rangle \\ |\psi_2\rangle &= c_{21} |\psi_1^{(0)}\rangle + c_{22} |\psi_2^{(0)}\rangle \end{aligned}$$

where the $|\psi_n^{(0)}\rangle$ are the eigenstates of $H^{(0)}$. I derived in class that

$$c_{11} = \frac{1}{\sqrt{2}} \left(1 + \frac{\Delta}{\sqrt{\Delta^2 + V^2}} \right)^{1/2} \quad \text{and} \quad c_{21} = \frac{1}{\sqrt{2}} \left(1 - \frac{\Delta}{\sqrt{\Delta^2 + V^2}} \right)^{1/2}$$

for $\Delta = H_{22} - H_{11}$ assumed greater than zero, and $H'_{12} = \frac{1}{2}V \exp(i\phi)$. In the course of that solution, we also obtained

$$c_{12} = c_{11} \exp(-i\phi) \left(\frac{\Delta - \sqrt{\Delta^2 + V^2}}{V} \right) \quad \text{and} \quad c_{22} = c_{21} \exp(-i\phi) \left(\frac{\Delta + \sqrt{\Delta^2 + V^2}}{V} \right)$$

(a) Complete the derivation to obtain the final formulas for c_{12} and c_{22} ,

$$c_{12} = -\frac{e^{-i\phi}}{\sqrt{2}} \left(1 - \frac{\Delta}{\sqrt{\Delta^2 + V^2}} \right)^{1/2} \quad \text{and} \quad c_{22} = \frac{e^{-i\phi}}{\sqrt{2}} \left(1 + \frac{\Delta}{\sqrt{\Delta^2 + V^2}} \right)^{1/2}$$

for $\phi = \arg(H'_{12})$.

(b) For the case $\Delta \gg V$, Taylor expand each of the c_{nm} to first order in the elements of H' , and compare the correction terms to the general formula (6.12) in the text. Note my notation is slightly different from Griffiths', with my c_{nm} corresponding to his $c_m^{(n)}$. Also, you'll need to adjust the arbitrary overall phase of the $|\psi_2\rangle$ state to correct for the fact that $|\psi_2\rangle$ should really be identical to $|\psi_2^{(0)}\rangle$ when H' is vanishingly small.

(c) Determine the c_{nm} for the case $\Delta = 0$.