Lecture 39

Last time, talked a bit about wave function collapse
Claimed don't really need, get same result by letting
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|D\rangle \to \frac{1}{\sqrt{2}}(|0\rangle|D_0\rangle + |1\rangle|D_1\rangle)$$
where $|D\rangle$ = state of detector (whether micro or macro)

Quantum states of this kind are special
call them <u>entangled</u>

More precisely, say that two quantum systems are entangled when joint quantum state $\Psi(1,2)$ is not a simple product $\phi_a(1)\phi_b(2)$

For instance, problem 12.1 - Sangha

Entangled states are interesting
Say $\psi = \frac{1}{\sqrt{2}}(\phi_a(1)\phi_b(2) + \phi_b(1)\phi_a(2))$

Neither 1 nor 2 in definite state

Say put 2 on rocket, send to distant star
Then measure particle 1 ... what happens?

Conventional: ψ collapses $\psi \to \phi_a(1)\phi_b(2)$ or $\phi_b(1)\phi_a(2)$

Happens instantaneously, even if 2 is light years away

Many worlds: $\psi \to \frac{1}{\sqrt{2}}(\phi_a(1)\phi_b(2)D_a + \phi_b(1)\phi_a(2)D_b)$

Again, instantaneous global change in ψ

Eitherway, seems to violate special relativity
Typically bothers people

Two comforting facts:

1) Relativity says nothing can travel faster than light
   if wave functions are not actual "things",
   no conflict

   Does raise question of what Ψ is, but we
   have no obligation to answer that

2) Don't violate causality

   Seems like it does:
   To moving observer, can look like Ψ collapses
   at particle 2 before measurement of 1
   But you can't see Ψ collapse

   Imagine trying to send message faster than light
   On earth, I measure 1
   Far away, you measure 2

   If I get ϕ_a, I immediate know you have ϕ_s
   But you can't tell Ψ has collapsed
   If you measure 2, get 50/50 ϕ_a or ϕ_s
   whether I have measured 1 or not

   Nice way to see:
   From your point of view, particle 1 already
   measured 2 long ago... could claim his
   Ψ has already collapsed, just not observed yet

So, no fundamental problem

But you might think that Ψ really is a physical thing & should obey special relativity.

Why not just assume that "collapse" propagates at speed of light c?

Run into trouble with Bell's theorem

Imagine particles are electrons, looking at spin states

As before, you and I are far apart
I have 1, you have 2

I'll measure spin along axis $\hat{a}$ ($\hat{x}, \hat{y}, \hat{z}$, or any other unit vector)

You measure along axis $\hat{b}$

For each, get either $\uparrow$ or $\downarrow$
Call $\uparrow = 1$, $\downarrow = -1$

Repeat many time, calculate correlation = average product

Run	Me	You	Product
1	1	−1	−1
2	1	−1	−1
3	−1	−1	1
4	1	−1	−1
5	−1	−1	1
6	−1	−1	1
		average =	0

Define average = $P(\hat{a}, \hat{b})$

$P(\vec{a},\vec{b}) > 0$, tend to get same answer
< 0 tend opposite
$= 0$ no correlation

Say initial state $\psi = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$

Find $P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b}$, problem 4.50

In particular, $P(\vec{a},\vec{a}) = -1$ for any $\vec{a}$

So if ψ is a real thing, must carry information about result of any possible measurement
Not "standard" wave function
write λ instead
sometimes called a "hidden variable"

So some function $A(\vec{a},\lambda)$ tells us what particle 1 will give when measured along $\vec{a}$
$= \pm 1$

and $B(\vec{b},\lambda)$ says what particle 2 gives when measured along $\vec{b}$

Note A is not same every time,
so λ must fluctuate or be uncertain

Describe by probability $\rho(\lambda)$

Then from normal probability rules

$$P(\vec{a},\vec{b}) = \int \rho(\lambda)\, A(\vec{a},\lambda)\, B(\vec{b},\lambda)\, d\lambda$$

Saw that $A(\vec{a},\lambda) = -B(\vec{a},\lambda)$ for all $\vec{a}, \lambda$

So $P(\vec{a},\vec{b}) = -\int \rho(\lambda)\, A(\vec{a},\lambda)\, A(\vec{b},\lambda)\, d\lambda$

For some third vector $\vec{c}$, have

$$P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) = -\int \rho(\lambda)\left[A(\vec{a},\lambda)A(\vec{b},\lambda) - A(\vec{a},\lambda)A(\vec{c},\lambda)\right]d\lambda$$

Use $A(\vec{b},\lambda)^2 = 1$

$$P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) = -\int \rho(\lambda)\left[A(\vec{a})A(\vec{b}) - A(\vec{a})A(\vec{b})A(\vec{b})A(\vec{c})\right]d\lambda$$

$$= -\int \rho\, A(\vec{a})A(\vec{b})\left[1 - A(\vec{b})A(\vec{c})\right]d\lambda$$

Then $|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \le \int \rho(\lambda)\, |A(\vec{a})A(\vec{b})|\, |1 - A(\vec{b})A(\vec{c})|\, d\lambda$

But $|A(\vec{a})A(\vec{b})| = 1$

and $1 - A(\vec{b})A(\vec{c}) \geq 0$

So

$$|P(\vec{a},b) - P(a,c)| \le \int \rho(\lambda)\left[1 - A(\vec{b})A(\vec{c})\right]d\lambda$$

$\int \rho\, d\lambda = 1$, so

$$\boxed{|P(\vec{a},\vec{b}) - P(a,c)| \le 1 - P(\vec{b},\vec{c})}$$

For any a, b, c

QM and experiment says $P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b}$

Say $\vec{a} = \hat{x}$ $\vec{b} = \hat{y}$ $\vec{c} =$ 45° in between

$$P(\vec{a},\vec{b}) = 0$$
$$P(\vec{a},\vec{c}) = -\frac{1}{\sqrt{2}}$$
$$P(\vec{b},\vec{c}) = -\frac{1}{\sqrt{2}}$$

So $P(a,b) - P(a,c) = \frac{1}{\sqrt{2}} = 0.7$

$1 + P(b,c) = 1 - \frac{1}{\sqrt{2}} = 0.3$

0.7 is not < 0.3, so Bell's inequality violated

So assumption was wrong:
No local entity λ that determines measurement results

(Every thing breaks down if λ changes in response to measurements)

Conclude that either Ψ doesn't really exist, or that it changes in a non-local way