Lecture 17

Coming up:
Rest of week: Zeeman effect
Next week: break
Monday 3/10: take home exam
Due Wednesday.

Covers material from Chs 4, 5, 6
- Finish & review Ch 6 on Monday

Open book & notes, but work alone
no time limit, but shouldn't be too long
(will post last year's)

HW 7 not due until 3/21
Week after exam

But, HW 7 covers Ch 6 material
Recommend you work on it over break
I'll be here if you have questions

Back to physics:
Finished fine structure

Good example of how PT goes:
Step (1) Work out H'
(2) Determine good basis to use
(3) Evaluate $\langle \psi_n | H' | \psi_n \rangle$

In general, none of these steps are easy

Zeeman effect is another example - shows how multiple perturbations combine

Put atom in magnetic field $\vec{B}$

$$H' = -\vec{\mu} \cdot \vec{B}$$

Generally $\vec{\mu}$ has two components

$$\vec{\mu}_S = -\frac{e}{m}\vec{S} \quad \text{from spin}$$
$$\vec{\mu}_L = -\frac{e}{2m}\vec{L} \quad \text{from orbit}$$

These just add: $H' = \frac{e}{2m}(\vec{L} + 2\vec{S}) \cdot \vec{B}$

Define coords so $\vec{B}$ along $\hat{z}$, get

$$\boxed{H'_z = \frac{e}{2m}(L_z + 2S_z) B}$$

Plain Bohr atom solutions = eigenstates of L_z, S_z

$$\text{so} \quad E^{(1)} = \frac{e\hbar}{2m}(m_\ell + 2m_s) B$$

But, fine structure breaks this!

Get 3 cases:

I) $H'_z \ll H'_{FS}$: Treat Zeeman as perturbation to FS states

II) $H'_{FS} \ll H'_z$: Treat FS as perturbation to Zeeman states

III) $H'_{FS} \simeq H'_z$: Treat together as perturbation to plain Coulomb

To decide which case, compare $\vec{B}$ to effective spin-orbit field

Recall B from "orbiting proton" $= \frac{1}{4\pi\varepsilon_0} \frac{e}{mc^2} \frac{1}{r^3} \vec{L}$

estimate $r \to a$, $\vec{L} \to \hbar$

plug in #'s, get $B_{proton} \approx 12$ Tesla

Pretty big, about as big as we can produce

So start with case $B \ll B_p$

Then use fine structure states, $|j\, m_j\rangle$ basis

Different m_j states are degenerate, but J_z commutes with L_z & S_z

So H'_z is diagonal w/in degenerate manifold

Just need to evaluate $E^{(1)} = \frac{eB}{2m} \langle j m_j | L_z + 2S_z | j m_j \rangle$

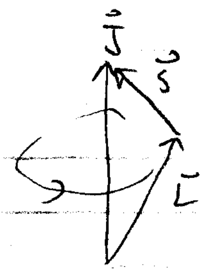
Have $J_z = L_z + S_z$ and $\langle J_z \rangle = m_j$

Just leaves $\langle j m_j | S_z | j m_j \rangle$

Working out for real is kind of a pain
Post derivation online

Easier to get using geometrical argument

Picture $\vec{L}$ & $\vec{S}$ precessing around fixed $\vec{J}$



Part of $\vec{S}$ that is $\perp$ to $\vec{J}$ is rapidly oscillating

$\rightarrow 0$ on average

Leaves only part of $\vec{S}$ $\parallel$ to $\vec{J}$

$$\vec{S}_{\parallel} = (\vec{S}\cdot\hat{J})\,\hat{J} = \frac{\vec{S}\cdot\vec{J}}{J^2}\,\vec{J}$$

Then $\langle S_z \rangle \rightarrow \langle S_{\parallel,z} \rangle = \langle j m_j | \frac{\vec{S}\cdot\vec{J}}{J^2} J_z | j m_j \rangle$

$$= \frac{1}{\hbar}\,\frac{m_j}{j(j+1)}\,\langle \vec{S}\cdot\vec{J} \rangle$$

We know how to handle $\vec{S}\cdot\vec{J}$:

$$\vec{L} = \vec{J} - \vec{S}$$
$$L^2 = J^2 + S^2 - 2\vec{S}\cdot\vec{J}$$

So $\vec{S}\cdot\vec{J} = \frac{1}{2}\left(J^2 + S^2 - L^2\right)$

$$\langle \vec{S}\cdot\vec{J} \rangle = \frac{\hbar^2}{2}\left[j(j+1) + s(s+1) - \ell(\ell+1) \right]$$

So get

$$\boxed{E^{(1)} = \frac{eB}{2m}\langle J_z + S_z \rangle = \frac{e\hbar B}{2m}\, m_j \left[1 \pm \frac{j(j+1) - \ell(\ell+1) + \frac{3}{4}}{2j(j+1)} \right]}$$

Typically write $E^{(1)} = \mu_B \, g_J \, m_j \, B$

$$\mu_B \equiv \frac{e\hbar}{2m} = \text{"Bohr magneton"}$$

$$= 9.27 \times 10^{-24} \; J/T$$

g_J = "Landé g-factor" = [ ]'s above

Problem 6.21

Analyze Zeeman effect for $n=2$ states

Get g_J's: $j=\frac{1}{2},\ l=0 \Rightarrow g_J = 2$

$j=\frac{1}{2},\ l=1 \Rightarrow g_J = \frac{2}{3}$

$j=\frac{3}{2},\ l=1 \Rightarrow g_J = \frac{4}{3}$

So shifts for each state are

j	l	m_j	$E^{(1)}/\mu_B B = g_J m_j$
$\frac{1}{2}$	0	$\frac{1}{2}$	$+1$
$\frac{1}{2}$	0	$-\frac{1}{2}$	-1
$\frac{1}{2}$	1	$\frac{1}{2}$	$+\frac{1}{3}$
$\frac{1}{2}$	1	$-\frac{1}{2}$	$-\frac{1}{3}$
$3/2$	1	$3/2$	$+2$
"		$1/2$	$+2/3$
"		$-1/2$	$-2/3$
"		$-3/2$	-2