Practice with power series expansions

It is hard to overestimate the importance of being able to make quick and accurate power series expansions. Skill at this will serve you well in all your physics classes. Here are ten problems you can use for practice. Try them for real before you look at the answers!

Each expression is to be expanded for small x. If not otherwise indicated, expand to *leading order*. That means you need to find the first non-zero term of the expansion that actually includes x; a constant zero-order term doesn't count.

You should be able to do the first seven in less than five minutes each. The last three have a little more room for mistakes, though they also can be done quickly with the right tricks.

1. $\sin(a + x)$
2. $\dfrac{1}{a^2 + x^2}$
3. Expand to order x^2 : $(a + x)^{5/2}$
4. $\dfrac{a + x}{\sqrt{b + x}}$
5. $e^{x \sin x}$
6. $\ln\left[1 + \ln(1 + x)\right]$
7. $\dfrac{x}{e^{\sqrt{a^2 + \sin^2(bx)}}}$
8. Expand to order x^2 : $\sqrt{1 + ax + bx^2}$
9. $\cos x + \dfrac{1}{2}\sin^2 x - 1$
10. Expand to order x^3 : $\left(1 - \sqrt{1 + x^2}\right)^{1/2}$