Power series expansion answers

1. \( \sin(a) + x \cos(a) \)
   
   (Either take the derivative, or expand the sine using the angle-sum formula and then take \( \cos x \to 1, \sin x \to x \).)

2. \( \frac{1}{a^2} - \frac{x^2}{a^4} \)

   (Use \( 1/(1 + \epsilon) \approx 1 - \epsilon \), with \( \epsilon = x^2/a^2 \).)

3. \( a^{5/2} \left( 1 + \frac{5}{2} \frac{x}{a} + \frac{15}{8} \frac{x^2}{a^2} \right) \)

   (Factor out the \( a \), and expand \((1 + \epsilon)^{5/2}\) by taking derivatives.)

4. \( \frac{1}{\sqrt{b}} \left[ a + \left( 1 - \frac{a}{2b} \right) x \right] \)

   (Factor out the \( b \), expand the denominator using \( \sqrt{1 + \epsilon} \approx 1 + \epsilon/2 \), then use \( 1/(1 + \epsilon) \approx 1 - \epsilon \), and finally multiply by \( a + x \), keeping the first order terms.)

5. \( 1 + x^2 \)

   (Use \( \sin x \approx x \), and then \( e^\epsilon \approx 1 + \epsilon \).)

6. \( x \)

   (Apply \( \ln(1 + x) \approx x \) twice.)

7. \( x e^{-|a|} \)

   (To leading order, can ignore the \( x \)-dependence in the denominator.)

8. \( 1 + \frac{ax}{2} + \left( \frac{b}{2} - \frac{a^2}{8} \right) x^2 \)

   (Expand \( \sqrt{1 + \epsilon} \) to second order by taking derivatives. In first order term, use \( \epsilon = ax + bx^2 \). In second order term, use \( \epsilon = ax \).)

9. \( -\frac{x^4}{8} \)

   (Either expand the trig functions to fourth order, or use trig relations to simplify expression to \( -(1 - \cos x)^2/2 \) and then expand the cosine to second order.)

10. \( \frac{1}{\sqrt{2}} \left( |x| - \frac{|x|^3}{8} \right) \)

    (Expand the inner square root to fourth order in \( x \). Factor \( x^2 \) out of the resulting expression, and then expand the square root of what is left to second order. The fact that you get \( |x| \) here means that you can’t just take derivatives to get the answer.)