Graphing Tutorial

Generating a Bode plot from a given transfer function is not trivial, since it involves isolating and plotting the magnitude and phase. Even if the math is right, it is important for the plot itself to be presentable and clear. This tutorial will step you through the creation of a typical, if complicated, Bode plot. The instructions are for using Excel, and you may want to refer to the Excel appendix in your lab manual as you proceed. If you prefer, you are welcome to use a different program, but you will need to figure out for yourself how to make the plots as described.

We will start with the transfer function

$$G = \frac{1 - x^2}{4 + ix - x^2},$$

where x represents a dimensionless scaled frequency, like $\omega\tau$. The magnitude of G can be determined by taking the magnitudes of the numerator and denominator, since the magnitude of a ratio is the ratio of the magnitudes. This yields

$$|G| = \frac{|1 - x^2|}{\sqrt{x^2 + (4 - x^2)^2}}.$$

The phase is most easily determined by inverting G and using

$$\phi = -\tan^{-1}\left(\frac{\mathsf{Im}(1/G)}{\mathsf{Re}(1/G)}\right)$$

yielding

$$\phi = -\tan^{-1}\frac{x}{4 - x^2}.$$

If you are not comfortable with these derivations, I'll be happy to step through them with you. We would like to plot these functions from $x = -0.01$ to $x = 100$.

First, we need to create a suitable range of x values. To get a decent plot that conveys all the information, we should use a large number of points, between say 20 and 100. Since we are making a log plot, we would like points with $\log(x)$ evenly spaced. Let us use column A for the x values. Enter 0.01 in cell A1, and then in cell A2 enter `=A1*1.2`. Drag-fill the formula down to row 52, which should then contain the value 109.2. Replace that last value with 100.

Column B will contain the $|G|$ values. Enter the formula
        `=20*LOG10(ABS(1-A1^2)/SQRT(A1^2+(4-A1^2)^2))`
and drag fill the column down to the bottom. Select columns A and B, and insert a new chart. The chart type should be XY (Scatter), and choose the lines without markers option. (There are too many points here to make markers attractive.) Note that you should not use smoothed lines, because they can be misleading. If you feel like you don't have enough points for the Straight Lines option to look nice, generate more data!

Select the horizontal axis and make it a log scale, and set the range to 0.01 to 100 if necessary. Observing the plot, you should see an abrupt-looking feature near $x = 1$. Make sure you understand all such discontinuities: here you can see that $G \to 0$ at $x = 1$, so a discontinuity in the log plot makes sense.

To make the plot look nicer, format the horizontal axis and set the Value axis Crosses option to 0.01. Similarly, format the vertical axis to make the horizontal axis cross at -35. This way the axis lines don't interfere with the presentation of your data. Then add some labels. Add a horizontal axis title "log(x)" and a vertical axis title "$|G|$ (dB)".

For the phase values, use column C. Enter the formula

```
=-180/PI()*ATAN(A1/(4-A1^2))
```

and drag-fill it. Generate a second scatter plot, and set the horizontal axis to a log scale. Here, you observe a discontinuity at $x = 2$ which bears examination. From the original function, we see that $G(2) = 1.5i$, and for x near 2, G should be close to $1.5i$. This corresponds to a phase of $+90°$, so evidently the phase for $x < 2$ was incorrect. This is because the arctan function is only defined up to $\pm 180°$. To correct for it, we can manually add $+180$ to the values in column C for $x < 2$.

However, this is a particularly tricky function! It is clear that $G(0) = 1$, with a phase of $0°$, not $+180°$. Evidently, another correction is required somewhere between $x = 0$ and $x = 2$. From the form of G, the obvious place to look is near $x = 1$. As x passes through 1, G changes sign, which corresponds to a $180°$ phase offset. Our arctan formula misses this, so we must put it in by hand. In the end, then, we need to add $180°$ to all the phase values between $x = 1$ and $x = 2$, and leave the rest alone. The resulting plot has a discontinuity at $x = 1$, but we have established that to be correct. Finish up the phase plot by adjusting the axis positions and adding axis titles, with the vertical title here reading "phase (degrees)".

Finally, let's suppose we have some measured data points we'd like to compare to our theory, listed below. Add these points to your spreadsheet in columns D, E, and F. To compare easily with the theory curve, add them to the same plots, using the Source Data option. Since the points are sparse, change them to be plain markers, with no lines or curves. When adding the new data sets, name them "Data" and rename the original data sets "Theory." This will set the label in the plot legend correctly. Note that after adding these points, you will need to readjust the axis crossing positions.

Finally, add a title to both plots saying "Tutorial Bode Plot." Then print out the plots. The easiest way to print both plots on one page is to use the Print Area function and select just the graphs. You can use Print Preview to make sure this worked correctly. You may use the printer in the room 219 lab if you want. Turn in your printout with your assignment.

x	$|G|$ (dB)	ϕ (deg)
0.01	-12	0
0.1	-12	0
1	-45	-52
2	4	80
10	0	8
100	0	0