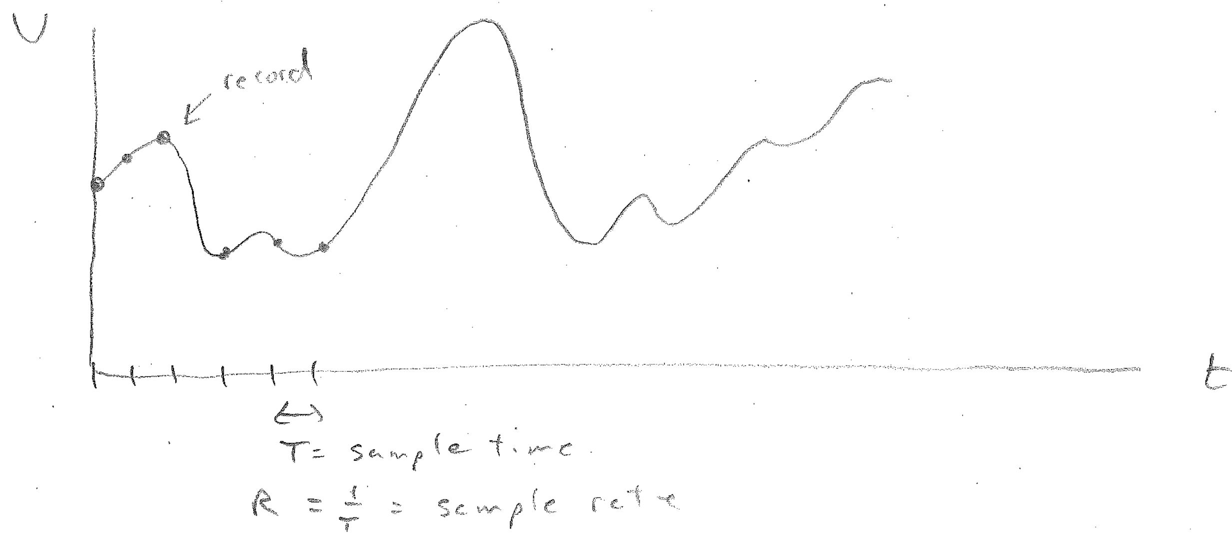
Lecture 24 Nyquist Sampling Theorem

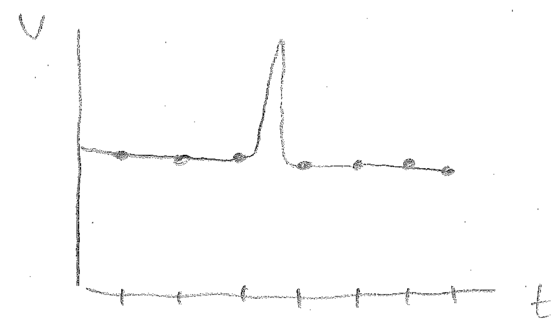
Earlier saw Nyquist stability criterion for servos

Another result: Nyquist sampling theorem

Applies when you are sampling a time-varying waveform
= recording discrete points in wave



Pretty obvious that sampled data misses information
if $V(t)$ varies faster than R:



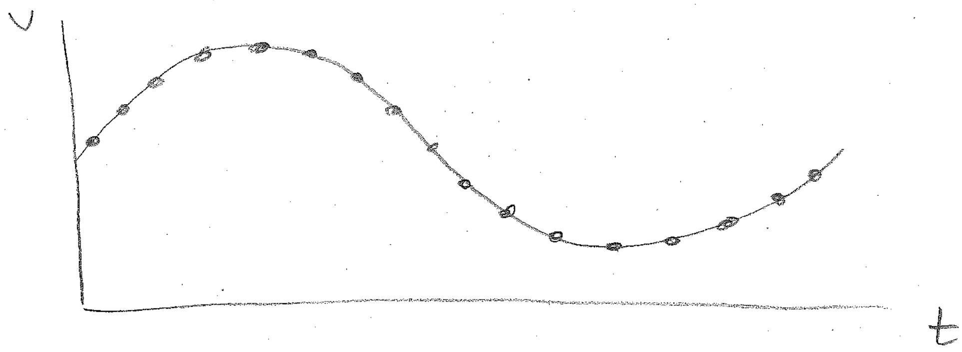
Nyquist makes this quantitative:

If $V(t)$ contains frequency components up to f_{max},
Then it can be perfectly reconstructed from samples taken at $R = 2f_{max}$

Conversely: if $V(t)$ has freq. components greater than $R/2$, then sampling at R will introduce errors

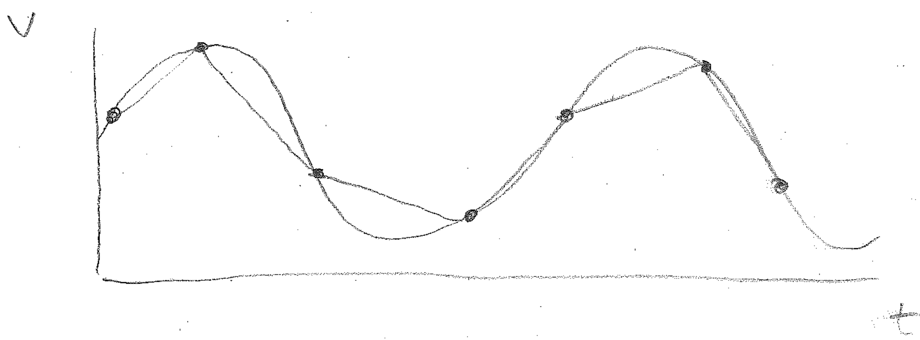
To understand, consider sine wave signals at freq f

$R \gg f$:



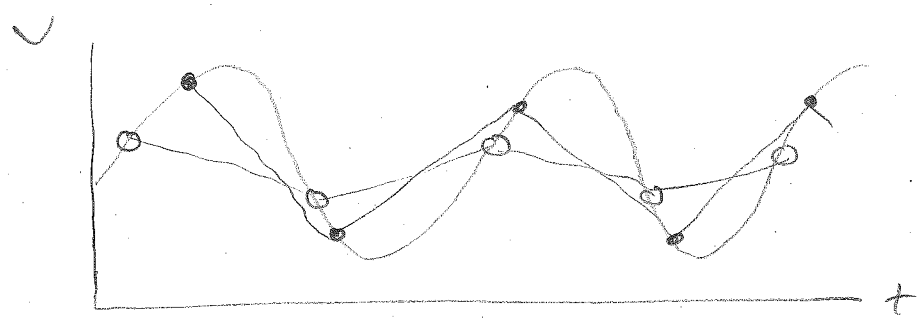
Works fine

$R > 2f$



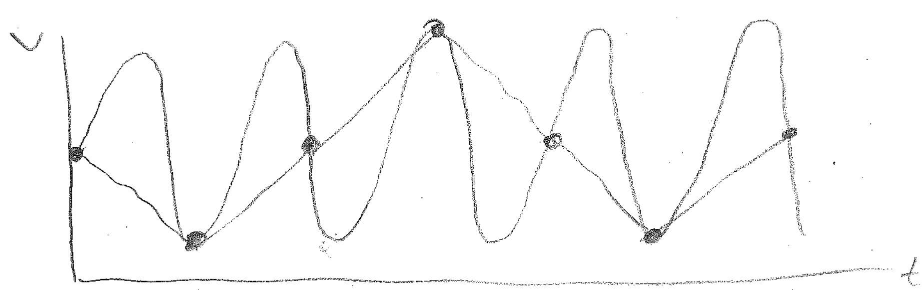
Looking just at sampled values, can see oscillation at f. A fit to a sine wave would be accurate

$R = 2f$



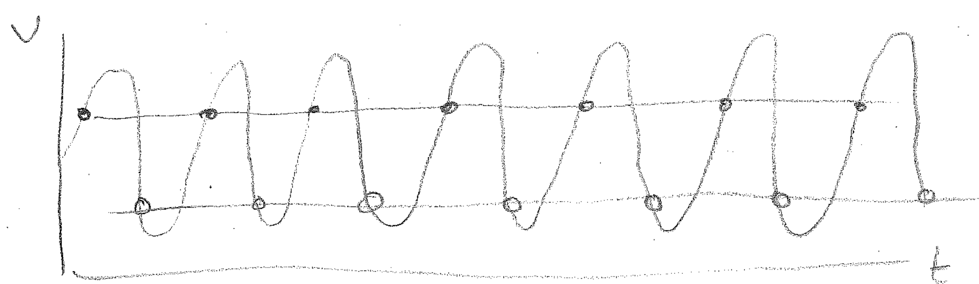
See oscillation at f, but observed amplitude depends on phase of samples

$R < 2f$



Look like oscillation at new freq $< f$

$R = f$

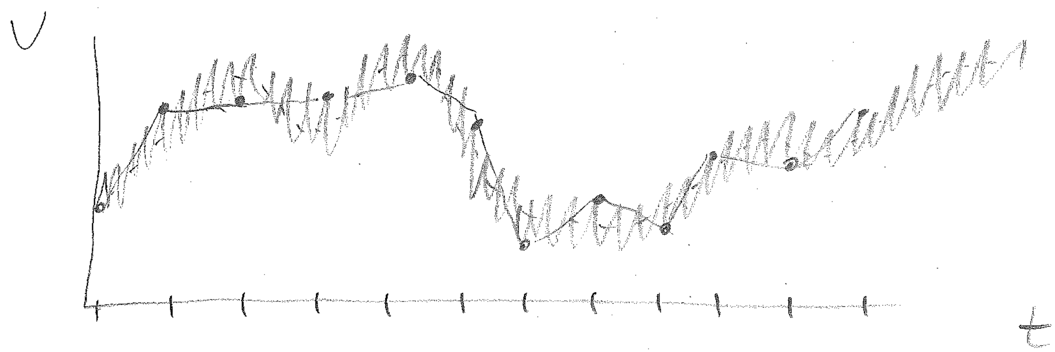


Get flat line

In fact, if $f > R/2$, sampled signal "looks" like wave at $f' = R - f$

Called aliasing: undersampled frequencies show up as lower frequency components

Aliasing is bad: might not care about high frequency "noise", but it distorts low freq signal



Proper fix:

Any time you sample a signal, use a low pass filter to attenuate components faster than $R/2$

(If you want to see faster components, need to increase R)

In real life, things tend to look more complicated

In lab, test by digitizing a signal, then reconstituting it with DAC

Should be able to see basic idea.

Note: Nyquist theorem important for communications

If channel has freq bandwidth f_{max}, max communication rate is $R = f_{max}/2$