Lecture 23  Analog/Digital conversion

Real world signals are generally analog
Need to convert analog to digital for measurements "A/D"
convert digital to analog for control "D/A"

Many methods.

Simplest: comparator

\[ \text{Like op-amp without feedback} \]
\[ \text{If } U_+ > U_- \quad V_{out} = +\text{supply} \]
\[ \text{If } U_- < U_+ \quad V_{out} = -\text{supply} \]

Make supply voltages +5V, 0V:
get one-bit ADC
Or make \( U = 2.5V \), supplies = desired analog levels
get one-bit DAC

Often, one bit is sufficient
Just need yes/no answer
or do/don't command

When more than one bit is required, use fancier techniques
DAC = digital to analog converter
Common method: \( R-2R \) ladder

\[ \begin{align*}
\text{Vref} & \quad R \quad R \quad R \quad 2R \\
2R & \quad 2R \quad 2R \quad 2R \quad 2R \\
\text{FET switch,} & \quad \text{controlled} \quad \text{by one binary bit} \\
\text{fixed} & \quad \text{reference} \\
\text{voltage} & \quad \text{virtual} \\
& \quad \text{grounded}
\end{align*} \]

\[ \text{N copies} \quad U_{out} \]
Input network acts like:

\[ V_{\text{ref}} \quad V_N \quad \frac{R}{2} \quad V_3 \quad \frac{R}{2} \quad V_2 \quad \frac{R}{2} \quad V_1 \quad \frac{2R}{N} \]

**Compare** \( V_2 = V_1 \)

\[ \begin{align*}
V_2 & = V_1 \\
2R & \quad \frac{R}{2} \\
V_1 & \quad \frac{R}{2} \\
P & \quad 2R \quad \text{(effective)}
\end{align*} \]

**Compare** \( V_3 = V_2 \)

\[ \quad \frac{R}{2} \quad \frac{R}{2} \quad \frac{R}{2} \quad \frac{R}{2} \]

\[ V_3 = \frac{V_2}{2} \]

Same for each stage: \( V_{n+1} = 2V_n \quad n = 1 \text{ to } N-1 \)

\[ = 2^n V_1 \]

So \( V_N = 2^{N-1} V_1 \), but \( V_N = V_{\text{ref}} \)

\[ \Rightarrow V_1 = \frac{V_{\text{ref}}}{2^{N-1}} \quad V_n = \frac{V_{\text{ref}}}{2^{N-n}} \]

Now, op amp is configured as summing amplifiers with gain \( \frac{1}{2} \)
Say \( b_n \) = bit value for \( n \)th stage

\[ b_n = 0 \text{ or } 1 \]

Then \( U_{out} = \frac{1}{2} \times \sum_{n=1}^{N} b_n \cdot U_n = \frac{1}{2} \sum_{n=1}^{N} b_n \cdot \frac{V_{ref}}{2^{N-n}} \]

\[ = \frac{V_{ref}}{2^N} \sum_{n=1}^{N} b_n \cdot 2^{N-1} \]

Now \( \sum b_n 2^n \) is just binary value of \( E b_n \)

\[ b_1 = 1^\text{st place} \]
\[ b_2 = 2^\text{nd place} \]
\[ b_3 = 4^\text{th place} \]
\[ \text{etc.} \]

So \( U_{out} > \Delta V \cdot \text{value } E b_n \)

\[ \Delta V = \frac{V_{ref}}{2^N} = \text{minimum voltage step} \]

So if \( N = 10 \) (values = 0 to 1023)

and \( \Delta V = 1 \mu \text{V} \), can get output value from

0 to 1.023 V

as specified by code.

Typical values are \( N = 8, 12, 16 \)

\[ V_{ref} = 1 \text{ to } 10 \text{ V} \]

\( \Delta U = \text{voltage resolution} \)

\( \text{typically } 1 \text{ to } 10 \mu \text{V} \)
ADC = analog to digital converter

measure voltage \( V \), output binary values \( \frac{V}{\Delta V} \)

with value \( \frac{v_{ref}}{\Delta V} = \frac{V}{\Delta V} \)

Larger \( N \) \( \Rightarrow \) more bits \( \Rightarrow \) smaller \( \Delta V \) \( \Rightarrow \) more resolution

Three common techniques:

- **Flash**: accurate & very fast, but limited to small \( N \)
  - Often used for digital scopes

- **Integrating**: very accurate but slow, works at large \( N \)
  - Often used for digital voltmeters

- **Successive Approximation**: fairly fast & fairly accurate
  - \( \Rightarrow \) medium \( N \)
  - Common general purpose method

Technique:

- Comparator
  - "successive approximation register"

\[ \text{Vin} \rightarrow \text{Comparator} \rightarrow \text{SAR} \rightarrow \text{DAC} \rightarrow \text{Output Register} \rightarrow \text{Digital out} \]

SAR generates one bit at a time

Initial output = middle of range = "guessed" for right value

DAC converts to voltage in middle of range

Comparator says whether guess is high or low

If guess is high, SAR rises to middle of upper half
If guess is low, SAR drops to middle of lower half

repeat \( N \) times
Clock controls timing for new guesses.
Register holds output until conversion complete.
Each cycle gives one more bit.

Example: \( N = 8 \) \( V_{\text{ref}} = 5 \text{ V} \) \( \Rightarrow \Delta V = \frac{5}{2^8} = 19.5 \text{ mV} \)

Say \( V_{\text{in}} = 1.330 \text{ V} \)

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Guess</th>
<th>Value</th>
<th>( \Delta V )</th>
<th>Comparator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000 0000</td>
<td>128</td>
<td>2.496 V</td>
<td>H</td>
</tr>
<tr>
<td>2</td>
<td>0100 0000</td>
<td>64</td>
<td>1.248 V</td>
<td>L</td>
</tr>
<tr>
<td>3</td>
<td>0110 0000</td>
<td>96</td>
<td>1.872 V</td>
<td>H H</td>
</tr>
<tr>
<td>4</td>
<td>0101 0000</td>
<td>80</td>
<td>1.560 V</td>
<td>H</td>
</tr>
<tr>
<td>5</td>
<td>0100 1000</td>
<td>72</td>
<td>1.404 V</td>
<td>H</td>
</tr>
<tr>
<td>6</td>
<td>0100 0100</td>
<td>68</td>
<td>1.326 V</td>
<td>L</td>
</tr>
<tr>
<td>7</td>
<td>0100 0110</td>
<td>70</td>
<td>1.365 V</td>
<td>H</td>
</tr>
<tr>
<td>8</td>
<td>0100 0101</td>
<td>69</td>
<td>1.346 V</td>
<td>H</td>
</tr>
</tbody>
</table>

Final output either \( 0100 0101 \)
or \( 0100 0100 \), depending on circuit details.

Might worry: what if \( V_{\text{in}} \) changes during measurement?

Solve with "sample and hold" circuit

\[
\text{Vin} \quad \overset{\text{FET}}{\longrightarrow} \quad \overset{\text{switch}}{\text{R}} \quad \overset{\text{following}}{\longrightarrow} \quad \text{to ADC}
\]

When switch is open, voltage on ADC held ~ constant.

Important for all ADC techniques.