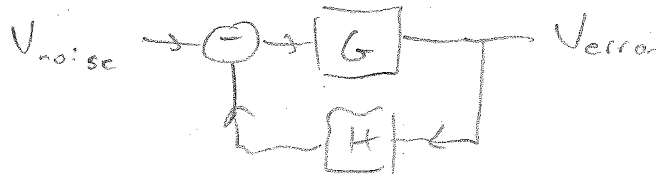
Lecture 14 - Improving Control

More on servos:



Get $V_{err} = \dfrac{G_0 V_{in}}{1 + G_0 H}$

Want $|1 + G_0 H| \gg 1$ to make V_{err} small

$\Rightarrow |G_0 H| \gg 1$

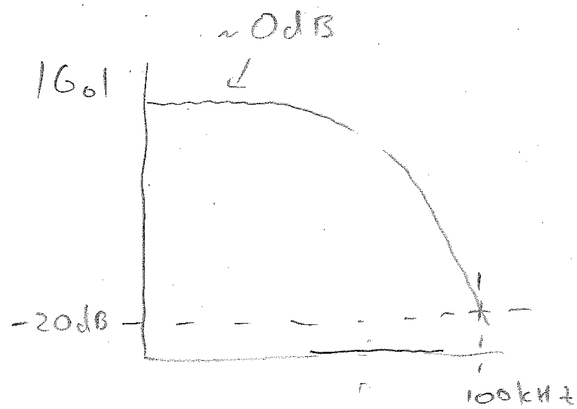
But as $\omega \to \infty$, $|G_0 H| \to 0$

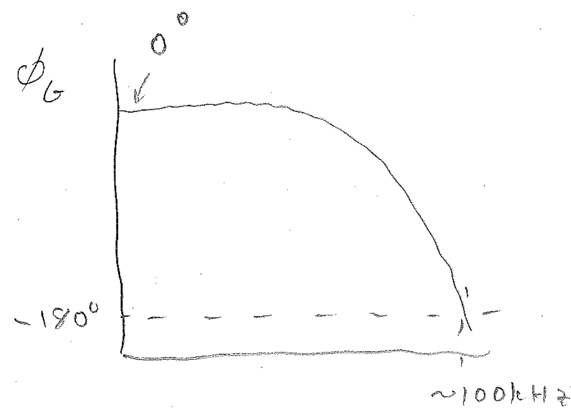
and $\phi_{GH} \to -\infty$

System is stable only if $|G_0 H| < 1$ when $\phi < -180°$

Limits how large H can be

From lab:





Ideally, H ~ constant and $\phi_H \sim 0°$

So $\phi_{GH} = \phi_G$

Define f_{180} = freq where $\phi_{GH} \to -180°$ $\quad f_{180} \approx 100\text{kHz}$

At 100 kHz, $|G_0| = -20\text{dB}$

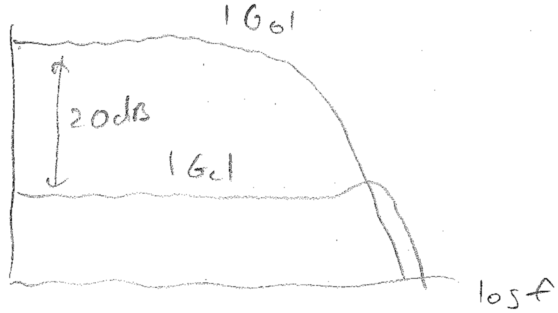
Need $|G_0 H| < 0\text{dB} \Rightarrow |H| < 20\text{dB}$

Larger H $\Rightarrow$ system starts to oscillate

Then at low frequencies, where $G_o \approx 1$,

noise reduction is $\dfrac{1}{1+G_oH} \approx \dfrac{1}{1+(1)\cdot(10)} \approx \dfrac{1}{11} \sim -20\,dB$

See



~ 90% noise reduction, not bad

But what if we want to do better?

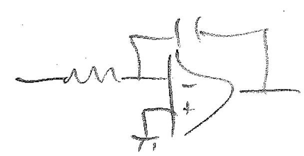
Key idea: make H frequency dependent

Ideally H large at low f, drops to 20 dB at f_{180}

(since we need $H(f_{180}) < 20\,dB$)

For instance, say $H \propto \frac{1}{f}$

Get from integrator



But integrator comes with $-90°$ phase:

$$H = \frac{1}{i\omega\tau} \qquad \tau = RC$$

$$\frac{1}{i} = e^{-i\pi/2}$$

We need $|G_oH| < 1$ before $\phi_{GH} = -180°$

$\phi_{GH} = \phi_G + \phi_H$ here $= \phi_G - 90°$

So we need to look at freq where $\phi_G = -90°$, $\equiv f_{90}$

Our circuit has $f_{90} \approx 20\,kHz$

$|G(f_{90})| \approx -10\,dB$

Still need $|G(f_{90})\,H(f_{90})| < 1$

$$-10\,dB + (H)_{dB} < 0\,dB$$

$$\Rightarrow \quad |H(f_{90})| < 10\,dB \sim 3\times$$

Have $|H| = \dfrac{1}{2\pi f \tau}$, so $H(f_{90}) = \dfrac{1}{2\pi f_{90}\cdot\tau} < 3$

So $\quad RC = \tau > \dfrac{1}{6\pi f_{90}} = 2.6\,\mu s$

As long as we obey this, system should be stable

But then noise reduction at <u>low</u> f is great!

At $f = 60\,Hz$, $\quad |H| = \dfrac{1}{2\pi\cdot 60\,Hz\cdot 2.6\,\mu s} = 1000 = 60\,dB$

So noise reduction is $\dfrac{1}{|1+G_0H|} = \dfrac{1}{|1+1\cdot(1000)|} \approx \dfrac{1}{1000}$

Factor of 100 better than before

Call this "integral control" ("I")

Version with $|H| = $ const called proportional control ("P")

Integral control pretty much just better

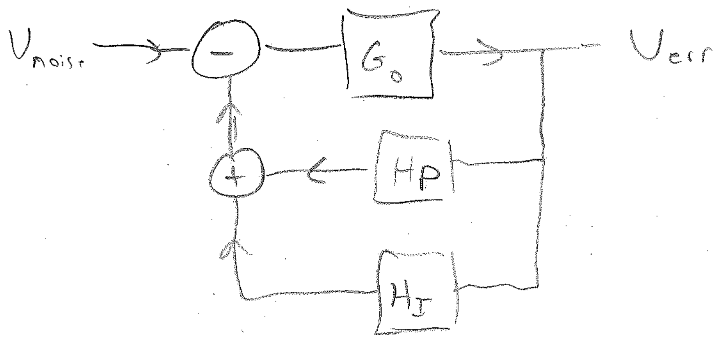
But! sacrifice some noise reduction at high frequencies

at f_{90}, I control has $|G_0H| < 1$

P control has $|G_0H| \gg 1$

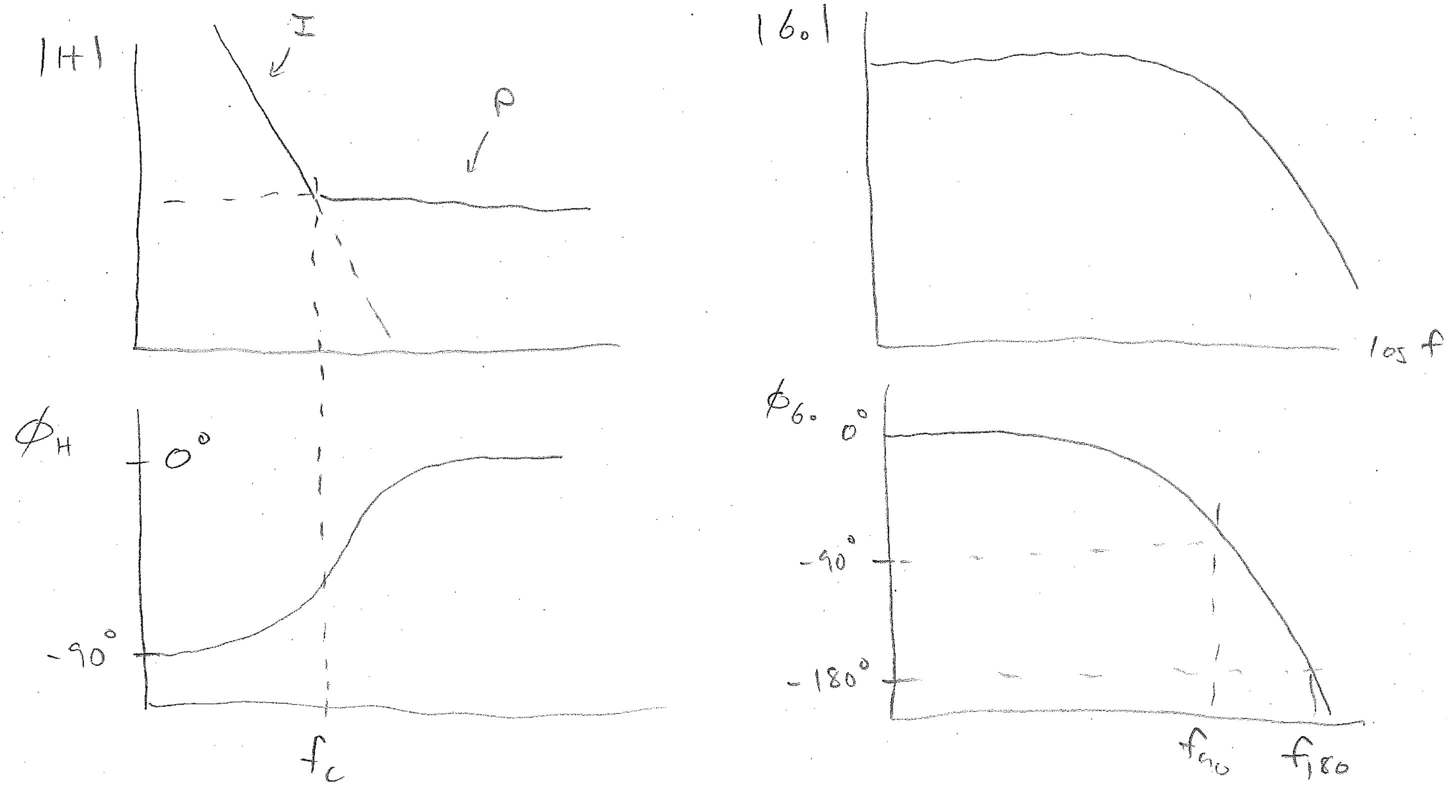
Usually, low frequencies more important

When high freqs do matter, get best of both:
use P + I together!



Called PI control

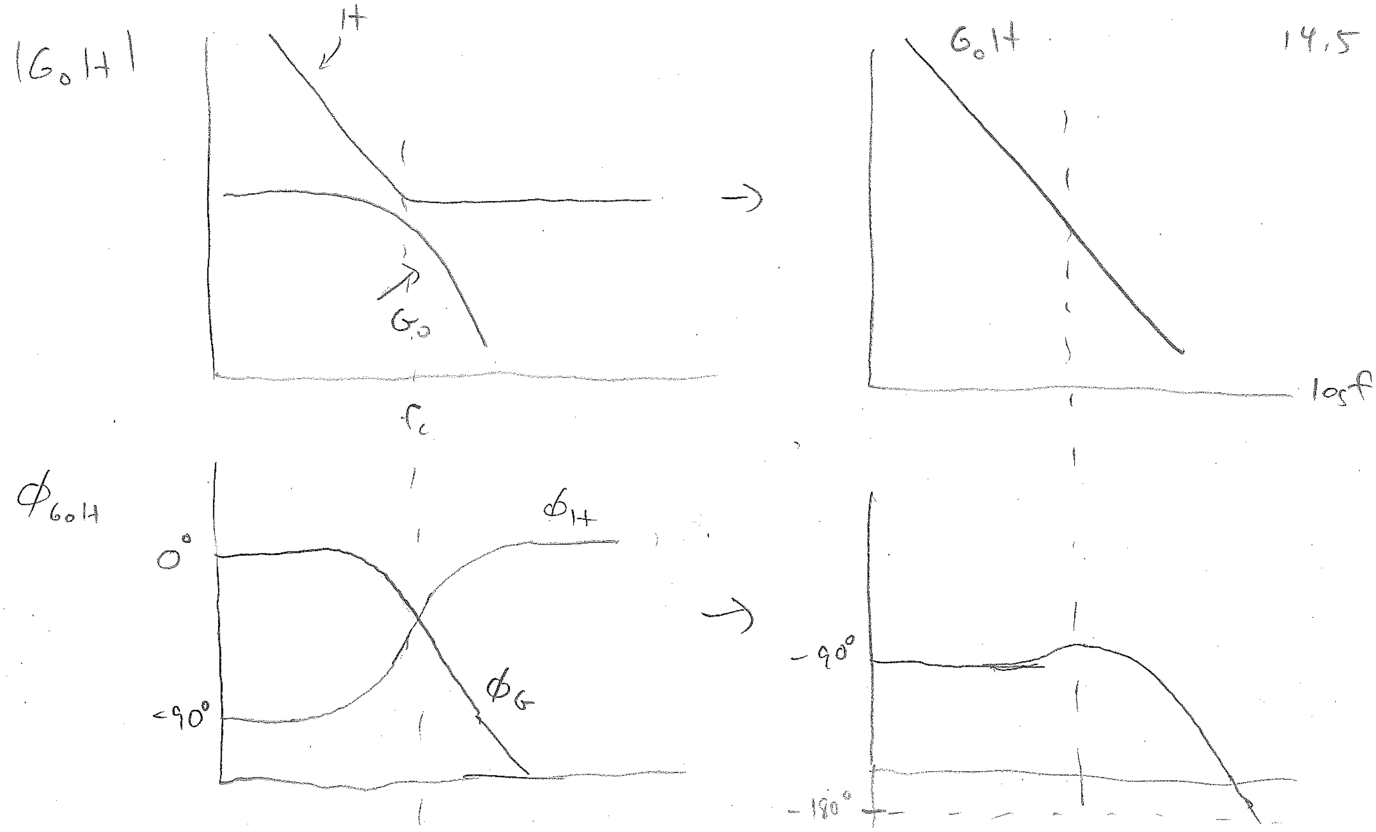
Hard to analyze algebraically, but graphs are useful



f_c = freq where P + I parts are equal magnitude
f_{90} = " where $\phi_G = -90°$
f_{180} = " where $\phi_G = -180°$

Say we make $f_c = f_{90}$

Then $|G_0 H|$



As ϕ_G decreases, ϕ_H increases to compensate

Total phase is $-180°$ at f_{180}, same as in P system

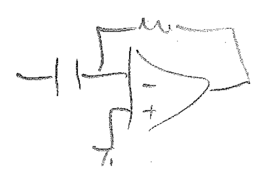
So at low freqs, get benefits of I

at high freqs, get benefits of P

Can extend idea further!

When ϕ_{G_0} hits $-180°$, increase ϕ_H again

Achieve with differentiator



$$H_D = i\omega RC$$

$$\phi = +90°$$

Add this to P&I, get PID controller

Works well if ϕ_G isn't changing too fast near $-180°$

But often ϕ_G drops fast! f_{270} close to f_{180}

Then derivative control doesn't help much