

Use electronic system to control a physical variable

Very general:

Position of a mass (using a motor)

Pressure in a tank (using a valve)

Current in a wire (using an amplifier)

Temperature in room (using heater)

etc

Same math applies to all

Set up: assume we have way to measure variable  
& to change it

Suppose variable subject to environmental noise

Use control system to eliminate/reduce noise

Example: Driving a car

Variable: position in lane

Measurement: visual

Control: steering wheel

Noise: bumps & turns in road

Strategy: Watch road. If car starts to move  
from center of lane, adjust wheel  
to correct

Want to automate this process in a circuit

System called "servomechanism" or "servo"

Useful to describe with block diagram

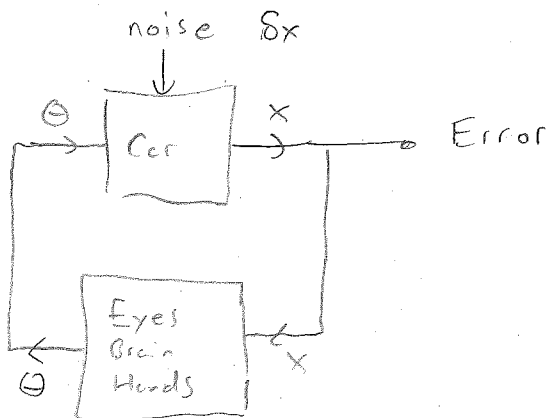
Each sub-system  $\rightarrow$  block with transfer function

(assume  $Z_{in}$ 's large and  $Z_{out}$ 's small)

Car Example:

$x$  = deviation from lane center

$\Theta$  = angle of steering wheel



Car described by  $G$ :  $x(\omega) = G(\omega)\Theta(\omega) + \delta x(\omega)$

Driver described by  $H$ :  $\Theta(\omega) = H(\omega)x(\omega)$

We want  $x = 0$ .

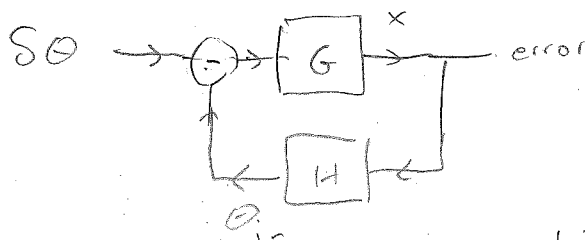
Define  $x$  = "error signal" = quantity we want to minimize

Note that  $\omega$ -dependence is essential

Neither car nor driver can respond to very fast inputs

Also, doesn't really matter where noise comes in

Could write:



Use  $\ominus$ : subtract two inputs

$$\Theta_{out} = S\Theta - \Theta_{in}$$

Why subtract? Know we want negative feedback. If car drifts right, steer left



Analyze:

$$V_{err} = G(V_{noise} - HV_{err})$$

$$V_{err}(1 + GH) = GV_{noise}$$

$$V_{err} = \frac{GV_{noise}}{1 + GH}$$

If  $H=0$  (no control), then  $V_{err} = GV_{noise}$

Call this = "open loop" response  $V_0$

Often write  $G = G_0 =$  open loop transfer function

With  $H \neq 0$ , call "closed loop" response  $V_c$

$$\text{So } V_c = \frac{V_0}{1 + G_0 H}$$

If  $|1 + G_0 H| \gg 1$ , then  $|V_c| \ll |V_0|$

control reduces noise in good!

Typically  $G_0$  set by system

But  $H$  is ours to implement.

Just make  $H$  huge?  $\rightarrow V_c \rightarrow 0$

No: problem with phase

Define loop phase  $\phi =$  phase of  $G_0 H = \phi_G + \phi_H$

Any physical system:  $\phi$  decreases as  $\omega \rightarrow \infty$

finite response time

At some  $\omega$ ,  $\phi \rightarrow -180^\circ$

$\Rightarrow$  sign of feedback  $\rightarrow$  positive!

Control increases noise, like microphone near speaker

Theorem (Nyquist stability criterion):

If  $|G_H| \equiv$  loop gain is greater than 1  
when  $\phi \leq -180^\circ$ , then system is unstable

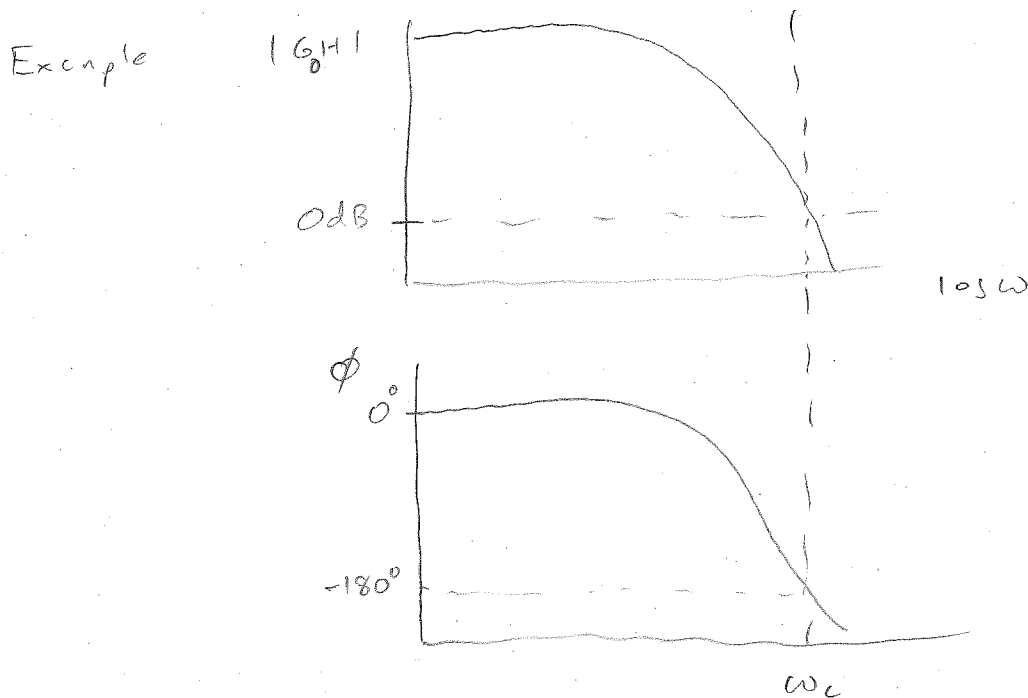
$$\frac{1}{1+GH} \rightarrow \infty \text{ at some } \omega$$

$\Rightarrow$  system freely oscillates at that  $\omega$

This would be bad, worse than no control!

Only fix: make sure  $|G_H|$  drops below 1  
before  $\phi$  reaches  $-180^\circ$ ,

This limits gain at lower freqs too!



Can't make H any larger without violating  
stability criterion.

Note: by convention, phase  $\phi$  doesn't count  
subtraction that makes feedback negative

12.5

Basically, assume  $\phi \rightarrow 0^\circ$  as  $\omega \rightarrow 0$

We'll spend two labs exploring effects, optimizing performance

Use simple system: photodiode illuminated by LED

Noise = ambient lights

compensate for ambient changes by adjusting LED