Lecture 12  
Servo Systems

Use electronic system to control a physical variable

Very general:
- Position of a mass (using a motor)
- Pressure in a tank (using a valve)
- Current in a wire (using an amplifier)
- Temperature in room (using heater)

etc

Some math applies to all

Set up: assume we have way to measure variable & to change it

Suppose variable subject to environmental noise
Use control system to eliminate/reduce noise

Example: Driving a car

Variable: position in lane
Measurement: visual

Control: steering wheel

Noise: bumps or turns in road

Strategy: Watch road. If car starts to move from center of lane, adjust wheel to correct

Want to automate this process in a circuit
System called "servomechanism" or "servo"
Useful to describe with block diagram.
Each sub-system as block with transfer function
(assume \( z_{in} \)'s large and \( z_{out} \)'s small)

Case Example:
\[ x = \text{deviation from lane center} \]
\[ \theta = \text{angle of steering wheel} \]

\[
\begin{align*}
\theta & \downarrow \\
\text{Car} & \rightarrow x \\
\text{Eyes} & \leftarrow x \\
\text{Brain} & \leftarrow x \\
\end{align*}
\]

Car described by \( G \):
\[ x(\omega) = G(\omega) \theta(\omega) + 8x(\omega) \]

Driver described by \( H \):
\[ \theta(\omega) = H(\omega)x(\omega) \]

We want \( x = 0 \)
Define \( x \) = "error signal" = quantity we want to minimize.

Note that co-dependence is essential.
Neither car nor driver can respond to very fast inputs.

Also, doesn't really matter where noise comes in.
Could write:

\[ S \Theta \rightarrow G \rightarrow x \rightarrow \text{error} \]

Use \( \Theta \): subtract two inputs
\[ \Theta_{out} = S\Theta - \Theta_{in} \]

Why subtract? Know we want negative feedback. If car drifts right, steer left.
General diagram:

![Diagram of control system](image)

Analyze:
\[ V_{err} = G (V_{noise} - H V_{err}) \]
\[ V_{err} (1 + GH) = GV_{noise} \]
\[ V_{err} = \frac{GV_{noise}}{1 + GH} \]

If \( H = 0 \) (no control), then \( V_{err} = GV_{noise} \)

Call this = "open loop" response \( V_0 \)

Often write \( G = G_0 = \text{open loop transfer function} \)

With \( H \neq 0 \), call "closed loop" response \( V_c \)

So \( V_c = \frac{V_0}{1 + GH} \)

If \( |1 + GH| \gg 1 \), then \( |V_c| \ll |V_0| \)

control reduces noise in good!

Typically \( G_0 \) set by system

But \( H \) is ours to implement.

Just make \( H \) huge? \( \Rightarrow V_c \to 0 \)

No! problem with phase

Define loop phase \( \phi = \text{phase of } GH = \phi_G + \phi_H \)

Any physical system \( \phi \) decreases as \( \omega \to \infty \)

finite response time
At some $\omega$, $\phi = -180^{\circ}$

$\Rightarrow$ sign of feedback is positive.

Control increases noise, like microphone near speaker.

Theorem (Nyquist stability criterion):

If $|G_j\omega| \leq 1$, loop gain is greater than 1 when $\phi \leq -180^{\circ}$, then system is unstable.

\[ \frac{1}{1 + G_j\omega} \rightarrow \infty \text{ at some } \omega \]

A system freely oscillates at that $\omega$.

This would be bad, worse than no control!

Only fix: make sure $|G_j\omega|$ drops below 1 before $\phi$ reaches $-180^{\circ}$.

This limits gain at lower freqs too!

Example $|G_j\omega|$:

\[ \text{Decibels} \]

$\phi$ $0^{\circ}$ $-180^{\circ}$ $\omega_c$

Can't make $H$ any larger without violating stability criterion.
Note: by convention, phase $\phi$ doesn't count subtraction that makes feedback negative

Basically, assume $\phi = 0^\circ$ as $\omega \to 0$

We'll spend two less exploring effects, optimizing performance

Use simple system: photodiode illuminated by LED

Noise = ambient light

Compensate for ambient changes by adjusting LED