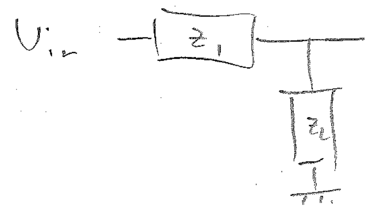
# Lecture 3 - Impedence & Transfer Functions

Recall voltage divider:

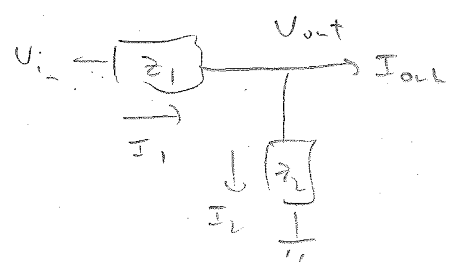



$$V_{OUT} = V_{in} \cdot \frac{Z_2}{Z_1 + Z_2}$$

This assumes no current flows from the output: $I_{out} = 0$

What if $I_{out} \neq 0$?

Can solve:




Have

$$I_1 = I_{out} + I_2$$

$$V_{in} - V_{out} = I_1 Z_1 \quad \rightarrow \quad V_{in} - V_{out} = \left(I_{out} + \frac{V_{out}}{Z_2}\right) Z_1$$

$$V_{out} = I_2 Z_2$$

$$V_{in} - I_{out} Z_1 = V_{out}\left(1 + \frac{Z_1}{Z_2}\right)$$

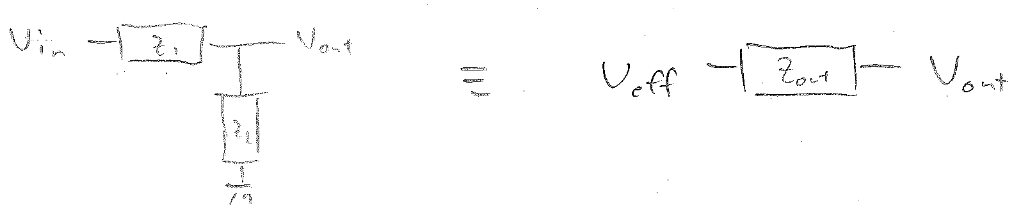
$$V_{OUT} = \frac{Z_2}{Z_1 + Z_2} V_{in} - I_{out} \frac{Z_1 Z_2}{Z_1 + Z_2}$$

So as $I_{out}$ increases, $V_{OUT}$ decreases.

This is a generic effect. Can describe with another version of Thevenin's Theorem:

Any network of sources and impedences acts like a single source $V_{eff}$ in series with single impedence $Z_{out}$

Here:



Equivalent circuit is easy to analyze:

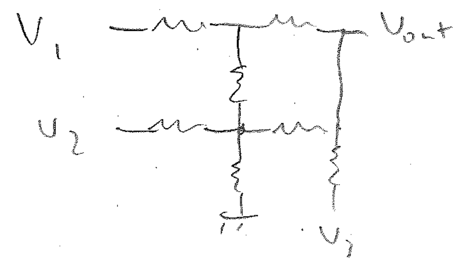
$$V_{out} = V_{eff} - I_{out} Z_{out}$$

Compare to above, see

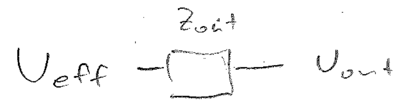
$$V_{eff} = V_{in} \cdot \frac{Z_2}{Z_1 + Z_2}$$

$$Z_{out} = \frac{Z_1 Z_2}{Z_1 + Z_2} = Z_1 \parallel Z_2$$

This is useful because it applies to any circuit




$\equiv$




$\Rightarrow \quad V_{out} = V_{eff} - I_{out} Z_{out}$

So for any circuit, output voltage decreases with current

$$Z_{out} = -\frac{dV_{out}}{dI_{out}} = \text{"output impedance"}$$

If you know $Z_{out}$, can say how much current can be supplied by a circuit.

Good power supply has low $Z_{out}$

How to determine $Z_{out}$:

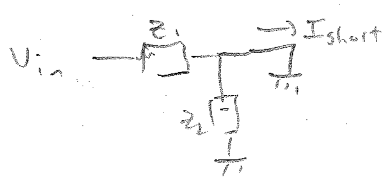
1) Assume some $I_{out}$, and solve for $V_{out}$, like we did with divider.

2) Easier: Assume $I_{out} = 0$ and solve for $V_{out} = V_{eff}$

Then assume $V_{out} = 0$ and solve for $I_{out}(\text{short})$

Then $Z_{out} = V_{eff} / I_{out}(\text{short})$

Example:




$$I_{short} = \frac{V_{in}}{Z_1}$$

$$\Rightarrow Z_{out} = \frac{\left(V_{in} \cdot Z_2/(Z_1+Z_2)\right)}{V_{in}/Z_1} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

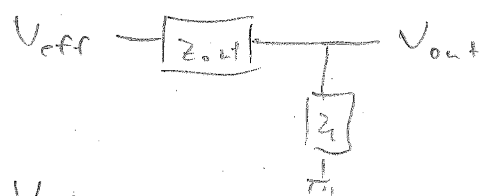
3) Good way to measure:

Measure $V_{eff} = V_{out}$ using voltmeter

Hook output to ground through impedance $Z_L$,

measure $V_{out}$ again

Effective circuit:




See $V_{out} = \dfrac{Z_L}{Z_{out} + Z_L} V_{eff}$

Solve for $Z_{out} = Z_L \times \dfrac{V_{out}}{V_{eff} - V_{out}}$

Some notes:

- Really only applies to linear circuits. Many circuits are nonlinear. But can still approximate as linear for (relatively) small $I_{out}$

- If $I_{out}$ is large, circuit dissipates internal power $I^2 R_{out}$. If too large, circuit can break! Try not to short outputs to ground unless you know circuit can survive.

Related topic: Input Impedance

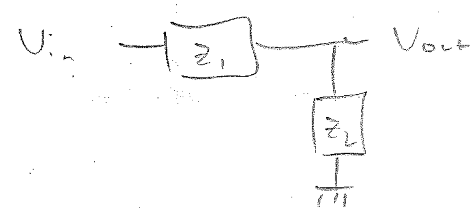
Output of any circuit acts like $V_{eff}$ —$\boxed{Z_{out}}$— $V_{out}$

Input of any circuit acts like $V_{in}$ —$\boxed{Z_{in}}$— $V_{float}$ )

($V_{float}$ = voltage present at input if you don't hook anything up.
(Generally $\neq V_{eff}$ (out)))

Very often, $V_{float}$ = ground

Example:




With no $V_{in}$ source, have $V_{in} = V_{out} = 0$
$\Rightarrow V_{float} = 0$

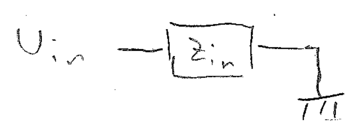
With $V_{in}$, have $I_{in} = \dfrac{V_{in}}{Z_1 + Z_2}$

$$\Rightarrow Z_{in} = Z_1 + Z_2$$

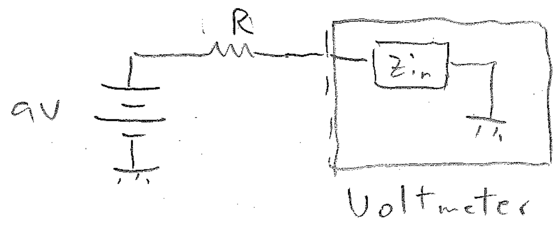
Note we assume $I_{out} = 0$, because no load is present.
If we had a load $Z_L$, $Z_{in}$ would change.

Input impedance important for measuring instruments

Voltmeter:




Describes fact that some current flows in to device. Can effect measurement:

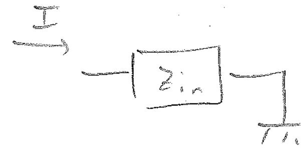



Forms divider, actually measures

$$V_{meas} = 9V \times \frac{Z_{in}}{Z_{in} + R}$$

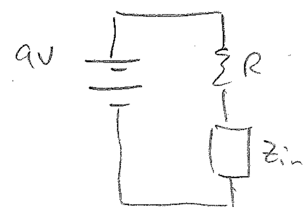
Ideally, $Z_{in} \gg R$, so $V_{meas} \to 9V$

Or in ammeter:




Needs some input voltage $V_{in} = I Z_{in}$ to operate

For instance:




$$I = \frac{9V}{R + Z_{in}}$$

Unless $Z_{in} \ll R$, $I$ changes when you add ammeter to circuit.

---

Introduce one more topic, discuss more next time: transfer function

Often have relation $V_{out} = (\quad) V_{in}$

divider: $V_{out} = \dfrac{Z_2}{Z_1 + Z_2} V_{in}$

In general, write $V_{out} = G V_{in}$

$G = G(\omega)$ depends on frequency

Generally complex

Call $G$ = transfer function

Describes what a circuit does

Often useful to express $G$ in polar form $G = |G| e^{i\phi}$

Show using two plots: $|G|$ vs $\omega$

$\phi$ vs $\omega$

But $|G|$ and $\omega$ typically vary a lot: use log plots

$\log |G|$ vs $\log \omega$ (or $\log f$)

$\phi$ vs $\log \omega$

Call pair: Bode plot

Convenient way to descibe how circuit functions

log|G| so convenient, it has a special unit! dB

write $g(\text{in dB}) = 20 \log |G|$

So $20 \text{ dB} \Rightarrow |G| = 10$

$-20 \text{ dB} \Rightarrow |G| = 0.1$

Usually see Bode plot as $g$ vs $\log_{10} f$

$\phi(\text{deg})$ vs $\log_{10} f$