Phys 3150/5190 — Homework Assignment 6

Due Wednesday, October 17

Note: If you need help with the complex math on this assignment, please be sure to come see me.

1. Verify the complex calculation used in Lab 7: If $G_0 = ge^{i\phi_G}$, $H = he^{i\phi_H}$, $z = gh$ and $\phi_z = \phi_G + \phi_H$, show that

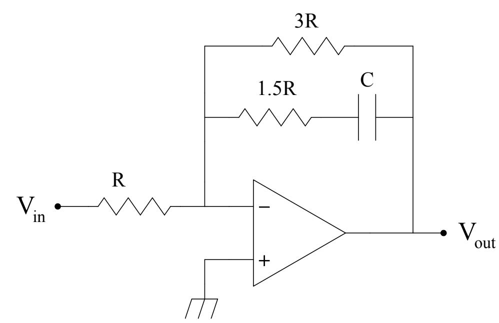
$$\left|\frac{G_0}{1+G_0H}\right| = \frac{g}{\sqrt{(1+z\cos\phi_z)^2 + (z\sin\phi_z)^2}}$$

and

$$\arg\left(\frac{G_0}{1+G_0H}\right) = \phi_G - \tan^{-1}\left(\frac{z\sin\phi_z}{1+z\cos\phi_z}\right).$$

(Recall $\arg q$ is the complex phase of q.)

2. Calculate the transfer function $G(\omega)$ for the circuit shown. (You can take the op amp behavior to be ideal.) Express your answer in terms of $\tau = RC$. Plot the gain (in dB) and phase as a function of $\log(\omega\tau)$. This and equivalent circuits are sometimes referred to as phase-lag compensators, and they can be useful for optimizing servo systems. *Hint:* when you have a transfer function G of the form A/B, you can use $\phi_G = \phi_A - \phi_B$.



3. Suppose you are designing a servomechanism for a system with open-loop transfer function

$$G_0 = \frac{-i\omega\tau_0}{1+i\omega\tau_1}$$

and with a feedback transfer function

$$H = \frac{A}{1+i\omega\tau_2}$$

with A and all the τ's real and positive.

(a) Determine the frequency ω_{180} where the loop phase reaches $-180°$. *Hint:* at this frequency, G_0H is real.

(b) Using the Nyquist criterion, determine the largest value of A for which the closed-loop system will be stable.