Phys 3150/5190 Complex Numbers Practice

1. If $q_1 = 2 + 3i$ and $q_2 = 1 - i$, evaluate the following:
   (a) $q_1 + q_2$
   (b) $q_1 q_2$
   (c) q_1/q_2
   (d) $|q_1| + |q_2|$
   (e) $|q_2/q_1|$

2. For polar form $q = |q|e^{i\phi}$, express $|q|$ and ϕ for the following:
   (a) $2 + 3i$
   (b) $1 - i$
   (c) $-3i$
   (d) -6

3. If the following are polar forms $(|q|, \phi)$, express q in Cartesian form $q = x + iy$:
   (a) $(1, 45°)$
   (b) $(3, -90°)$
   (c) $(2, 115°)$
   (d) $(0, 60°)$

4. For real x, y, and z, reduce the following expressions to Cartesian and polar forms:
   (a) $\frac{1}{x+iy}$
   (b) $\frac{x+iy}{x-iy}$
   (c) $\exp(x + iy)$
   (d) $\log(x + iy)$

Solutions

1.
(a) $4 + 2i$
(b) $2 - 2i + 3i + 3 = 5 + i$
(c) $(2 + 3i)(1 + i)/2 = (-1 + 5i)/2$
(d) $\sqrt{4 + 9} + \sqrt{1 + 1} = \sqrt{13} + \sqrt{2}$
(e) $|q_2/q_1| = |q_2|/|q_1| = \sqrt{2}/\sqrt{13}$

2.
(a) $|q| = \sqrt{13}, \phi = \tan^{-1}(3/2) = 56.31°$
(b) $|q| = \sqrt{2}, \phi = \tan^{-1}(-1/1) = -45°$
(c) $|q| = 3, \phi = \tan^{-1}(-3/0) = -90°$
(d) $|q| = 6, \phi = \tan^{-1}(-0/6) = 0°$

3.
(a) $x = 1\cos(45°) = 0.707, y = 1\sin(45°) = 0.707$
(b) $x = 3\cos(-90°) = 0, y = 3\sin(-90) = -3$
(c) $x = 2\cos(115°) = -0.845, y = 2\sin(115°) = 1.813$
(d) $x = 0 = y$

4.
(a)

$$\frac{1}{x + iy} = \frac{x - iy}{x^2 + y^2}, \quad \left|\frac{1}{x + iy}\right| = \frac{1}{\sqrt{x^2 + y^2}}, \quad \phi = -\tan^{-1}(y/x)$$

(b)

$$x + iy = re^{i\phi}, \quad r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}(y/x)$$
$$x - iy = re^{-i\phi} \quad \text{so} \quad \frac{x + iy}{x - iy} = e^{2i\phi}$$
$$\text{and} \quad \frac{x + iy}{x - iy} = \frac{x^2 - y^2 + 2ixy}{x^2 + y^2}$$

(c)

$$e^{x+iy} = e^x e^{iy} = e^x(\cos y + i \sin y)$$

(d)

$$x + iy = re^{i\phi} \quad \text{so} \quad \log(x + iy) = \log(re^{i\phi}) = \log(r) + i\phi$$