Econ 409 practice test

This is a closed book test. Please write in the space provided, and continue on the back of the page if needed, but mark "Continued on Back" clearly, so that I do not miss it. You should allocate time so that you are able to **write something** for all all parts, as I will give partial credit. The points sum to 75, so if your time spent approximately matches the points on each question, you will be finished in 75 minutes, which is the time limit for this test. **Please show your work, but you don't have to provide reasons for obvious steps.**

1. (5 points) For y = f(x), provide the definition the elasticity of y with respect to x.

$El_x f(x) =$

2. (5 points) Please evaluate the determinant of the matrix, showing your work:

6	12	4	0
2	6	1	0
4	6	3	0
0	0	0	1

3. (5 points) Invert the matrix:

1	2	3
0	1	2
0	0	1

4. a) (10 points) Use **Cramer's rule** (showing your work) to find the derivative dy/dz for the 2 equation model, where g(x) and f(y) are increasing and continuously differentiable functions with f'(y) > 1 and g'(x) > 1:

$g(x) + y = z$

$x + f(y) = 2z$

b) Determine the sign of the dy/dz derivative (explain), or show why the assumptions given are not sufficient to determine the sign.

5. (5 points) Find the derivative with respect to x of:

$\exp(f(x)g(x))$

$A^{f(x)}$

$Ax^{10} + 1/e^{x}$

6. (5 points) Please provide a formal definition of what it means for n vectors, $\mathbf{x}_1$, $\mathbf{x}_2$, ... $\mathbf{x}_n$, to be linearly independent.

7. (5 points) A utility function, u(x), has a measure of risk aversion defined to be $-u''(x)/u'(x)$. The risk aversion measure may be constant or it may be a function of x. Determine whether risk aversion is increasing in x, constant, or decreasing in x for the following utility functions:

$u(x) = x^{1-r}$, where $0 < r < 1$, and $x > 0$

$u(x) = 1-e^{-rx}$

8. (10 points) A monopoly firm produces output Q at at a constant average cost, c, so total cost is cQ. This product is sold in a market with an inverse demand curve: $p = D(Q)$, with $D'(Q) < 0$ and $D''(Q) < 0$. Show the first-order condition for profit maximization, and show that the price elasticity of demand (i.e. the elasticity of quantity with respect to price) is greater than 1 in absolute value at the optimal quantity. (That is, you are to verify the statement: "A monopolist never operates on the inelastic portion of the demand curve.")

9. (5 points) State the first-order and second-order necessary conditions for the a relative maximum of the function f(x), where x can be any real number.

10. (5 points) Prove that $(AB)^{-1} = B^{-1}A^{-1}$ if the matrices A and B have inverses.

11. (5 points) A diagonal matrix is defined to be one with zeros everywhere except on the diagonal, i.e. $a_{ij} = 0$ if i not equal j. What are sufficient conditions for a diagonal matrix to be idempotent.

12. (10 points) Consider the function, $f(x) = (x-2)^4$, which has a stationary point at x = 2. Use a third-order Taylor expansion with a remainder term involving the fourth derivative of f(x) to show that this stationary point is a relative minimum. (If you have trouble, begin by writing the formula for the Taylor series expansion of a function f(x) around a point x_0, and then try to apply it to the present problem in which the first three derivatives of the function are zero when evaluated at the stationary point.