Chapter 14
Oscillations
Units of Chapter 14

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- Simple Harmonic Motion Related to Uniform Circular Motion
- The Simple Pendulum
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If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic. The mass and spring system is a useful model for a periodic system.
14-1 Oscillations of a Spring

We assume that the surface is frictionless. There is a point where the spring is neither stretched nor compressed; this is the equilibrium position. We measure displacement from that point \((x = 0\) on the previous figure).

The force exerted by the spring depends on the displacement:

\[
F = -kx.
\]
The minus sign on the force indicates that it is a restoring force—it is directed to restore the mass to its equilibrium position.

\( k \) is the spring constant.

The force is not constant, so the acceleration is not constant either.
14-1 Oscillations of a Spring

- **Displacement** is measured from the equilibrium point.

- **Amplitude** is the maximum displacement.

- **A cycle** is a full to-and-fro motion.

- **Period** is the time required to complete one cycle.

- **Frequency** is the number of cycles completed per second.
If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.
Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator (SHO).

Substituting $F = -kx$ into Newton’s second law gives the equation of motion:

$$\frac{d^2x}{dt^2} + \frac{k}{m} x = 0,$$

with solutions of the form:

$$x = A \cos(\omega t + \phi).$$
Substituting, we verify that this solution does indeed satisfy the equation of motion, with:

\[ \omega^2 = \frac{k}{m}. \]

The constants \( A \) and \( \phi \) will be determined by initial conditions; \( A \) is the amplitude, and \( \phi \) gives the phase of the motion at \( t = 0 \).
14-2 Simple Harmonic Motion

The velocity can be found by differentiating the displacement:

\[ v = \frac{dx}{dt} = \frac{d}{dt} \left[ A \cos(\omega t + \phi) \right] = -\omega A \sin(\omega t + \phi). \]

These figures illustrate the effect of \( \phi \):

[Diagram showing oscillatory motion with annotations for phase shift and period.]
Example:

Determine the phase constant $\varphi$ in the equation

$$x(t) = A \cos(\omega t + \varphi)$$

If, at $t=0$

a) $x = -A$

b) $x = 0$

c) $x = A$

d) $x = A/2$

e) $x = -A/2$

f) $x = A/\sqrt{2}$
Because $\omega = 2\pi f = \sqrt{k/m}$, then

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}, \]

\[ T = 2\pi \sqrt{\frac{m}{k}}. \]
Example:

The springs of a 1500-kg car compress 5.0 mm when its 68-kg driver gets into the driver’s seat. If the car goes over a bump, what will be the frequency of oscillations? Ignore damping.
The velocity and acceleration for simple harmonic motion can be found by differentiating the displacement:

\[ v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \]

\[ a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi). \]
14-3 Energy in the Simple Harmonic Oscillator

We already know that the potential energy of a spring is given by:

$$ U = -\int F \, dx = \frac{1}{2} k x^2. $$

The total mechanical energy is then:

$$ E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2. $$

The total mechanical energy will be conserved, as we are assuming the system is frictionless.
14-3 Energy in the Simple Harmonic Oscillator

If the mass is at the limits of its motion, the energy is all potential.

If the mass is at the equilibrium point, the energy is all kinetic.

We know what the potential energy is at the turning points:

\[ E = \frac{1}{2} k A^2. \]
The total energy is, therefore, \( \frac{1}{2} k A^2 \).

And we can write:

\[
\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2.
\]

This can be solved for the velocity as a function of position:

\[
v = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}},
\]

where \( v_{\text{max}}^2 = (k/m) A^2 \).
This graph shows the potential energy function of a spring. The total energy is constant.

\[ E = K + U = \frac{1}{2} k A^2 \]

\[ U(x) = \frac{1}{2} k x^2 \]
Example

An object of mass 2.7 kg is executing simple harmonic motion, attached to a spring with spring constant \( k = 280 \text{ N/m} \). When the object is 0.020 m from its equilibrium position, it is moving with a speed of 0.55 m/s.

a) Calculate the amplitude of the motion.

b) Calculate the maximum speed of the object.
Conceptual Example 14-8: Doubling the amplitude.

Suppose this spring is stretched twice as far (to $x = 2A$). What happens to (a) the energy of the system, (b) the maximum velocity of the oscillating mass, (c) the maximum acceleration of the mass?
If we look at the projection onto the \( x \) axis of an object moving in a circle of radius \( A \) at a constant speed \( v_M \), we find that the \( x \) component of its velocity varies as:

\[
 v = v_M \sqrt{1 - \frac{x^2}{A^2}}.
\]

This is identical to SHM.
A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.
In order to be in SHM, the restoring force must be proportional to the negative of the displacement. Here we have: $F = -mg \sin \theta$, which is proportional to $\sin \theta$ and not to $\theta$ itself.

However, if the angle is small, $\sin \theta \approx \theta$ and $\cos \theta \approx 1 - \theta^2/2$. 
14-5 The Simple Pendulum

Therefore, for small angles, we have:

\[ F \approx -\frac{mg}{L} x, \]

where \( x = L\theta \).

The period and frequency are:

\[ T = 2\pi \sqrt{\frac{L}{g}}, \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}. \]
So, as long as the cord can be considered massless and the amplitude is small, the period does not depend on the mass.
Example:

Find a formula for the maximum speed $v_{\text{max}}$ of a simple pendulum bob in terms of $g$, the length $l$, and the maximum angle of swing $\theta_{\text{max}}$. 
A physical pendulum is any real extended object that oscillates back and forth.

The torque about point O is:

\[ \tau = -mg \, h \, \sin \theta. \]

Substituting into Newton’s second law gives:

\[ I \frac{d^2 \theta}{dt^2} = -mg \, h \, \sin \theta. \]
For small angles, this becomes:

\[ \frac{d^2 \theta}{dt^2} + \left( \frac{mgh}{I} \right) \theta = 0, \]

which is the equation for SHM, with

\[ \theta = \theta_{\text{max}} \cos(\omega t + \phi), \]

\[ T = 2\pi \sqrt{\frac{I}{mgh}}. \]
A student wants to use a meter stick as a pendulum. She plans to drill a hole through the meter stick and hang it from a smooth pin attached to a wall. Where in the meter stick should she drill the hole to obtain the shortest possible period? What is the shortest period of oscillation she can get?
Summary of Chapter 14 so far:

- A simple pendulum approximates SHM if its amplitude is not large. Its period in that case is:

\[ T = 2\pi \sqrt{\frac{L}{g}}. \]
A mass $M$ is attached to a spring with spring constant $k$. When this system is set in motion with amplitude $A$, it has a period $T$. What is the period if the amplitude of the motion is increased to $2A$?

A) $2T$
B) $T/2$
C) $T$
D) $4T$
E) $T$
Group questions:

A mass is attached to a vertical spring and bobs up and down between points A and B. Where is the mass located when its kinetic energy is a maximum?

A) at either A or B  
B) midway between A and B  
C) one-third of the way between A and B  
D) one-fourth of the way between A and B  
E) none of the above
The velocity of a mass attached to a spring is given by \( v = (1.5 \text{ cm/s}) \sin(\omega t + \pi/2) \), where \( \omega = 3.0 \text{ rad/s} \). What is the corresponding expression for \( x \)?

A) \( x = -(4.50 \text{ cm}) \sin(\omega t + \pi/2) \)
B) \( x = (4.50 \text{ cm}) \cos(\omega t + \pi/2) \)
C) \( x = -(0.50 \text{ cm}) \cos(\omega t - \pi/2) \)
D) \( x = -(0.50 \text{ cm}) \cos(\omega t + \pi/2) \)
E) \( x = -(0.50 \text{ cm}) \sin(\omega t + \pi/2) \)
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