Chapter 34
The Wave Nature of Light;
Interference
34-7 Luminous Intensity

The intensity of light as perceived depends not only on the actual intensity but also on the sensitivity of the eye at different wavelengths.

Luminous flux: 1 lumen = 1/683 W of 555-nm light

Luminous intensity: 1 candela = 1 lumen/steradian

Illuminance: luminous flux per unit area
Example 34-9: Lightbulb illuminance.

The brightness of a particular type of 100-W lightbulb is rated at 1700 lm. Determine (a) the luminous intensity and (b) the illuminance at a distance of 2.0 m.
Summary of Chapter 34

• The wave theory of light is strengthened by the interference and diffraction of light.

• Huygens’ principle: every point on a wave front is a source of spherical wavelets.

• Wavelength of light in a medium with index of refraction $n$:

$$\lambda_n = \frac{\lambda}{n}.$$

• Young’s double-slit experiment demonstrated interference.
Summary of Chapter 34

• In the double-slit experiment, constructive interference occurs when

\[ d \sin \theta = m\lambda, \quad m = 0, 1, 2, \ldots. \]

\[ \begin{bmatrix} \text{constructive} \\ \text{interference} \\ \text{bright} \end{bmatrix} \]

• and destructive interference when

\[ d \sin \theta = \left( m + \frac{1}{2} \right)\lambda, \quad m = 0, 1, 2, \ldots. \]

\[ \begin{bmatrix} \text{destructive} \\ \text{interference} \\ \text{dark} \end{bmatrix} \]

• Two sources of light are coherent if they have the same frequency and maintain the same phase relationship.
Summary of Chapter 34

• Interference can occur between reflections from the front and back surfaces of a thin film.

• Light undergoes a 180° phase change if it reflects from a medium of higher index of refraction.
Chapter 35
Diffraction and Polarization
Units of Chapter 35

• Diffraction by a Single Slit or Disk
• Intensity in Single-Slit Diffraction Pattern
• Diffraction in the Double-Slit Experiment
• Limits of Resolution; Circular Apertures
• Resolution of Telescopes and Microscopes; the \( \lambda \) Limit
• Resolution of the Human Eye and Useful Magnification
• Diffraction Grating
Units of Chapter 35

• The Spectrometer and Spectroscopy
• Peak Widths and Resolving Power for a Diffraction Grating
• X-Rays and X-Ray Diffraction
• Polarization
• Liquid Crystal Displays (LCD)
• Scattering of Light by the Atmosphere
If light is a wave, it will diffract around a single slit or obstacle.
35-1 Diffraction by a Single Slit or Disk

The resulting pattern of light and dark stripes is called a diffraction pattern.
This pattern arises because different points along a slit create wavelets that interfere with each other just as a double slit would.

\[ \sin \theta = \frac{\lambda}{D} \]

- \( \theta = 0 \) Bright
- \( \sin \theta = \frac{\lambda}{D} \) Dark
- \( \sin \theta = \frac{3\lambda}{2D} \) Bright
- \( \sin \theta = \frac{2\lambda}{D} \) Dark
35-1 Diffraction by a Single Slit or Disk

The minima of the single-slit diffraction pattern occur when

$$D \sin \theta = m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \ldots$$

[minima]
Example 35-1: Single-slit diffraction maximum.

Light of wavelength 750 nm passes through a slit 1.0 x 10^{-3} mm wide. How wide is the central maximum (a) in degrees, and (b) in centimeters, on a screen 20 cm away?
Diffraction by a Single Slit or Disk

Conceptual Example 35-2: Diffraction spreads.

Light shines through a rectangular hole that is narrower in the vertical direction than the horizontal. (a) Would you expect the diffraction pattern to be more spread out in the vertical direction or in the horizontal direction? (b) Should a rectangular loudspeaker horn at a stadium be high and narrow, or wide and flat?
Light passing through a single slit can be divided into a series of narrower strips; each contributes the same amplitude to the total intensity on the screen, but the phases differ due to the differing path lengths:

\[ \Delta \beta = \frac{2\pi}{\lambda} \Delta y \sin \theta. \]
35-2 Intensity in Single-Slit Diffraction Pattern

Phasor diagrams give us the intensity as a function of angle.

\[ E_0 (= N \Delta E_0) \]

At center, \( \theta = 0 \).

\[ \Delta E_0 \]

\[ \Delta E_0 \]

Between center and first minimum.

First minimum, \( E_\theta = 0 \) (\( \beta = 2\pi = 360^\circ \)).

Near secondary maximum.
35-2 Intensity in Single-Slit Diffraction Pattern

Taking the limit as the width becomes infinitesimally small gives the field as a function of angle:

\[ E_\theta = E_0 \frac{\sin \beta/2}{\beta/2}. \]
Finally, we have the phase difference and the intensity as a function of angle:

$$\beta = \frac{2\pi}{\lambda} D \sin \theta.$$  

and

$$I_\theta = I_0 \left(\frac{\sin \beta/2}{\beta/2}\right)^2.$$
35-2 Intensity in Single-Slit Diffraction Pattern

Example 35-3: Intensity at secondary maxima.

Estimate the intensities of the first two secondary maxima to either side of the central maximum.
35-3 Diffraction in the Double-Slit Experiment

The double-slit experiment also exhibits diffraction effects, as the slits have a finite width. This means the amplitude at an angle $\theta$ will be modified by the same factor as in the single-slit experiment:

$$E_{\theta 0} = 2E_0 \left( \frac{\sin \beta/2}{\beta/2} \right) \cos \frac{\delta}{2}.$$

The intensity is, as usual, proportional to the square of the field.
The diffraction factor (depends on $\beta$) appears as an “envelope” modifying the more rapidly varying interference factor (depends on $\delta$).
Example 35-4: Diffraction plus interference.

Show why the central diffraction peak shown, plotted for the case where $d = 6D = 60\lambda$, contains 11 interference fringes.
Resolution is the distance at which a lens can barely distinguish two separate objects.

Resolution is limited by aberrations and by diffraction. Aberrations can be minimized, but diffraction is unavoidable; it is due to the size of the lens compared to the wavelength of the light.
For a circular aperture of diameter $D$, the central maximum has an angular width:

$$\theta = \frac{1.22\lambda}{D}.$$ 

[$\theta$ in radians]
The Rayleigh criterion states that two images are just resolvable when the center of one peak is over the first minimum of the other.
Example 35-5: Hubble Space Telescope.

The Hubble Space Telescope (HST) is a reflecting telescope that was placed in orbit above the Earth’s atmosphere, so its resolution would not be limited by turbulence in the atmosphere. Its objective diameter is 2.4 m. For visible light, say $\lambda = 550$ nm, estimate the improvement in resolution the Hubble offers over Earth-bound telescopes, which are limited in resolution by movement of the Earth’s atmosphere to about half an arc second. (Each degree is divided into 60 minutes each containing 60 seconds, so 1° = 3600 arc seconds.)
Example 35-6: Eye resolution.

You are in an airplane at an altitude of 10,000 m. If you look down at the ground, estimate the minimum separation $s$ between objects that you could distinguish. Could you count cars in a parking lot? Consider only diffraction, and assume your pupil is about 3.0 mm in diameter and $\lambda = 550$ nm.
35-5 Resolution of Telescopes and Microscopes; the $\lambda$ Limit

For telescopes, the resolution limit is as we have defined it:

$$\theta = \frac{1.22\lambda}{D}.$$  \[\theta \text{ in radians}\]

For microscopes, assuming the object is at the focal point, the resolving power is given by

$$RP = s = f\theta = \frac{1.22\lambda f}{D}.$$
Example 35-7: Telescope resolution (radio wave vs. visible light).

What is the theoretical minimum angular separation of two stars that can just be resolved by (a) the 200-inch telescope on Palomar Mountain; and (b) the Arecibo radio telescope, whose diameter is 300 m and whose radius of curvature is also 300 m. Assume $\lambda = 550$ nm for the visible-light telescope in part (a), and $\lambda = 4$ cm (the shortest wavelength at which the radio telescope has been operated) in part (b).
Typically, the focal length of a microscope lens is half its diameter, which shows that it is not possible to resolve details smaller than the wavelength being used:

\[ \text{RP} \approx \frac{\lambda}{2}. \]
The human eye can resolve objects that are about 1 cm apart at a distance of 20 m, or 0.1 mm apart at the near point.

This limits the useful magnification of a light microscope to about 500x–1000x.
A diffraction grating consists of a large number of equally spaced narrow slits or lines. A transmission grating has slits, while a reflection grating has lines that reflect light.

The more lines or slits there are, the narrower the peaks.
Assignment:

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