Repeated Partnerships with Multiple Equilibria and Imperfect Monitoring: An Experimental Study

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Abstract

I investigate repeated partnership games with imperfect monitoring where both mutual effort and mutual shirking are Nash equilibria of the stage game. I find that period 1 effort rates are increasing in the number of repetitions, but subjects use trigger strategies that switch to permanent shirking after enough failed projects so effort rates decrease as the game progresses. Additionally, the rate of decrease is less when there are more repetitions. These results are consistent with a theory of strategic uncertainty in which a subject best responds to their beliefs about whether their partner exerts effort or shirks. Finally, I show that total effort does not vary much as the number of repetitions is increased (perhaps it slightly increases) because the increased period 1 effort is essentially canceled out by the erosion of effort as the game progresses.

1 Introduction

In the canonical application of repeated games, repetition provides a mechanism through which players can achieve efficient outcomes that are not possible in the stage game.[1] This is cleanly demonstrated by the classic prisoner's dilemma; defection (the inefficient action) is the unique Nash equilibrium of the prisoner's dilemma but cooperation (the efficient action) may be possible in a subgame perfect equilibrium

[1] I use the common term stage game for the game that is repeated.

of the repeated game. However, sometimes relationships sour, and when they do, it is easy to imagine a scenario in which repetition could negatively impact efficiency as the players could become stuck choosing non-cooperative and inefficient actions after the relationship deteriorates. The prisoner's dilemma that has been studied extensively cannot adequately address this scenario as inefficiency is guaranteed in the non-repeated game, so repetition can only help. In this paper, I consider a game that allows for, and arguably gives a very good shot for, decreased efficiency with repetition.

The two crucial features of the game are two pure Nash equilibria of the stage game and imperfect monitoring of actions. To understand why these features are critical, it is useful to go into the details of the game. The stage game is a two-player partnership game. The players simultaneously decide to exert effort or shirk on a project that can either succeed or fail. Effort is costly but if both players exert effort the probability of a success is increased. The parameters are such that mutual effort and mutual shirking are both Nash equilibria of the stage game, and mutual effort Pareto dominates but mutual shirking risk dominates (as defined by Harsanyi and Selten (1988)).[2] There are two key points here. First, multiple Nash equilibria is essentially a prerequisite for repetition to decrease efficiency. If there is a unique Nash equilibrium and it is inefficient, as in the prisoner's dilemma, then an inefficient outcome is guaranteed in the non-repeated game so repetition can only have a positive effect. On the other hand, if the unique Nash equilibrium is efficient, then it is technically possible to have decreased efficiency if the players support an inefficient outcome with intertemporal incentives, but, in practice this seems unlikely to occur as it would be simple and Pareto dominating to just repeat the Nash equilibrium in the repeated game. Second, the fact that the Pareto inferior equilibrium is risk dominant makes it attractive as well as otherwise there would be little reason to suspect anything but mutual effort in both the repeated and non-repeated game.

Still, to get decreased efficiency, the players must coordinate on effort in the non-repeated game and shirking in the repeated game. This brings me to the other crucial feature of the game; players observe the outcome of the project at the end of each repetition, but not the action of their partner. For a player who exerts effort, successes are more likely if their partner exerts effort and thus outcomes act as signals about their partner's action. Specifically, a success signals effort from their partner while a failure signals shirking. As the player wants to match the action of their partner,

[2] In fact, the expected payoffs of the stage game are such that the game is equivalent to the well-known stag hunt game.

perhaps a player starts the repeated game with effort but switches to shirking after observing one (or several) failures. If the game lasts long enough, a project will eventually fail and so perhaps failures will re-coordinate the two players on shirking at some period in the repeated game.

Another, perhaps more subtle, design feature of this game is that, for a player who shirks, the probability of success is independent of their partner's action and so the outcome of the project is an uninformative signal. So a player who shirks does not update their beliefs about their partner's action and thus shirking remains optimal in the next period and ultimately in every period after. That is, if the players get re-coordinated on shirking, they will not be able to re-coordinate again on effort.

Putting everything together, there could be effort in the non-repeated partnership game, and effort in the early periods of the repeated game but shirking in later periods. If this is the case, then there would be decreased efficiency if there are enough repetitions. However, there are two mitigating factors that predict more effort in the repeated game. They are both consequences of the subtle feature just mentioned that the outcome of the project is only an informative signal if a player chooses effort. This information is valuable, but only in the repeated game, and so there is an extra incentive to choose effort in the repeated game to obtain information. This means that period 1 effort rates (the proportion of subjects who choose effort) could be larger in the repeated game, and that players may be more hesitant to trigger shirking in longer repeated games.

In what follows, I first describe the partnership game and the repeated game in detail. I then formalize the previous discussion with a model of strategic uncertainty in which a player is uncertain whether their partner is exerting effort or shirking. Importantly, this is in contrast to a pure equilibrium model where each player knows exactly what the other player is doing and so the signals just described would be irrelevant. Such models of strategic uncertainty have been found to be highly predictive in the infinitely repeated prisoner's dilemma (see Dal Bó and Fréchette (2011)) where there is also multiple equilibria (and no communication).

I then investigate the predictions of the model and ultimately determine whether repetition decreases efficiency in a laboratory experiment. The treatment variable is the number of repetitions and is set to 1 (the non-repeated game) and 5, 10, and 25 (repeated games).[3] The predictions of the model are strongly supported. Trigger strategies are ubiquitous and so effort rates erode over time in the repeated games.

[3] I investigate a finitely repeated game even though most of the literature I contrast to is infinitely repeated games. I discuss why this is appropriate later.

There is also more period 1 effort and the rate of erosion is slower as the number of repetitions increases. These two factors work against each other for total effort and, for the most part, cancel out. In total, more repetition does not decrease efficiency (it possibly increases efficiency slightly although not in a statistically significant way).

There are two strands of related literature. First, the study of repeated games in the laboratory is extensive (see Dal Bó and Fréchette (2018) for a recent survey). The literature has mostly focused on the perfect monitoring infinitely repeated prisoner's dilemma, and similar games, where repetition can only increase efficiency (and the main finding is that it does although in fewer circumstances than theory predicts). Some studies have introduced imperfect monitoring (recent examples include Aoyagi and Fréchette (2009), Fudenberg, Rand, and Dreber (2012), and Embrey, Fréchette, and Stacchetti (2013)). Aoyagi and Fréchette (2009) do find slightly decreased efficiency if the imperfect signals are perfect noise although this is attributed to abnormally high rates of cooperation in the non-repeated prisoner's dilemma (about 35% as compared to 5% in Dal Bó (2005) and 0% in Duffy and Ochs(2009)) and so the result may have gone away if subjects had gained enough experience (they only had subjects play the game 7 times). Embrey, Fréchette, and Stacchetti (2013) also study a partnership game like the one in this paper although, crucially, the parameters are set so the stage game has the structure of a prisoner's dilemma.

Second, the partnership game is a coordination game with a tradeoff between risk and efficiency and these games have been examined quite thoroughly as well. Most related are the stag hunt game (see Cooper, DeJong, Forsyth, and Ross (1990)), which the partnership game is equivalent too although monitoring is perfect in the stag hunt, and the minimum (or median) effort game (see Van Huyck, Battalio, and Beil (1990)), which is a different game but does have imperfect monitoring (see Devetag and Ortmann (2007) for a survey of such games). While studies have not considered repeated versions of these games as I do here, many of them do use the fixed matching protocol which means the experiments are a single repeated game. A single repeated game has serious drawbacks, as a single mistake or misunderstanding of the instructions has no chance to be corrected, but the games are similar enough to warrant a comparison of results. In the stag hunt, Schmidt, Shupp, Walker, and Ostrom (2003) and Clark and Sefton (2001) compare fixed matching to random matching and both find more efficiency when matching is fixed so repetition increases efficiency. However, monitoring is perfect so more efficiency is not too surprising. In the minimum effort game, Berninghaus and Ehrhart (1998) use fixed matching and vary the number of repetitions and find that more repetitions increases efficiency as well. They also simultaneously lower the payoff per period which translates to lower cost of experimenting with risky and more efficient actions in early periods and this is the channel to which they attribute their result.

2 Theory

2.1 Partnership Game

The stage game for the experiment is a partnership game. There are 2 players who work on a project. Each player i simultaneously chooses an action $a_i \in A = \{e, s\}$ where e is effort and s is shirk. The outcome of the project is $o \in \{succ, fail\}$ where $succ$ is a success and $fail$ is a failure. The outcome is jointly determined by the actions of the players following the distribution

$$Prob(o = succ|a_1 = a_2 = e) = p \quad \text{and} \quad Prob(o = succ|a_1 = s \text{ or } a_2 = s) = q$$

where $1 > p > q > 0$. The cost of effort is c and the benefit of a success is z so player i's payoff is $\mathbb{1}_{o=succ}z - \mathbb{1}_{a_i=e}c$. Therefore, the game matrix with expected payoffs is

	e	s
e	$pz - c$, $pz - c$	$qz - c$, qz
s	qz, $qz - c$	qz, qz

I assume that $(p - q)z > c$ so there are two pure strategy Nash equilibria of the partnership game, (e, e) and (s, s). This assumption also implies that (e, e) Pareto dominates (s, s). Furthermore, I assume that $c > 1/2(p - q)$ so that (s, s) is the risk-dominant equilibrium (as defined by Harsanyi and Selten (1988)). The latter assumption is not critical from an equilibrium perspective, but creates a strategic tension between efficiency and risk.

The partnership game is repeated for T periods. In the repeated game, players observe the outcome o_t of the project at the end of each period t. Each player i does not observe player $j \neq i$'s action $a_{j,t}$. So, at the beginning of period t, the history $h_t^i = ((a_{i,1}, o_1), \ldots, (a_{i,t-1}, o_{t-1}))$ summarizes all the information player i has about the game so far. I denote the set of all period t histories by $\mathcal{H}_t$ (where $\mathcal{H}_1 = \emptyset$ by convention) and the set of all histories by $\mathcal{H} = \cup_{t=1}^{T} \mathcal{H}_t$. A (pure) strategy σ_i for player i is a mapping from $\mathcal{H}$ to A and a strategy $\sigma = (\sigma_1, \sigma_2)$ is a strategy for each player. I denote the set of all strategies for player i by $\mathcal{S}$. A subgame perfect equilibrium is a strategy σ such that $\sigma_i(h)$ is a best response to $\sigma_{-i}(h)$ in the repeated game starting from history h for every history $h \in \mathcal{H}$ for both $i = 1, 2$.

There are a lot of subgame perfect equilibria in this game. In particular, any sequence of Nash equilibria of the stage game is a subgame perfect equilibrium of the full T period game. This includes strategies such as *always effort*, choose e in every period; *always shirk*, choose s in every period; *grim trigger*, choose e until $fail$ and then s thereafter; *alternate*, choose e in odd periods and s in even periods; etc. [4]

2.2 Strategic Uncertainty and Trigger Strategies

There are multiple equilibria and no communication which suggests that it would be difficult to coordinate on a particular equilibrium. In this section, I relax the assumption of equilibrium and assume rather that player i forms a belief $\mu \in \Delta\mathcal{S}$ over the possible strategies of player $j \neq i$, and then best responds to this belief. This assumption has proven to be a much better predictor of behavior in the repeated prisoner's dilemma,[5] and thus I adapt it here to the repeated partnership game.

First, consider $T = 1$, the non-repeated partnership game. Suppose player i believes $j \neq i$ chooses e with probability ρ and s with probability $1 - \rho$. Then, e is the best response if and only if $(\rho p + (1 - \rho)q)z - c \geq qz$ which simplifies to $\rho \geq \frac{c}{(p-q)z}$.

Now, suppose $T > 1$. Here, I will show why optimal play is generally a trigger strategy: a strategy that may start the repeated game with effort but permanently switches to shirking if enough projects fail. Intuitively, when player i exerts effort, project failures signal that j is more likely shirking so enough fails will make shirking the best response. The switch is permanent, because, if a player shirks, the probability of failure is independent of the opponent's action and so no new information about their strategy is obtained and thus shirking remains optimal in future periods.

To formalize this argument, I focus on the simple case where player i believes j plays always effort with probability ρ_t and always shirk with probability $1 - \rho_t$ in period t. Let $r_{\rho_t} = Prob(o_t = succ | a_{i,t} = e, \rho_t) = \rho_t p + (1 - \rho_t)q$. In period T, i will choose effort if and only if it is myopically optimal given their period T belief ρ_T. This is the same analysis as for the $T = 1$ case above. That is, $a_{i,T} = e$ iff

[4] In fact, a pure strategy is a subgame perfect equilibria if and only if it is a sequence of Nash equilibria. The action profiles (e, s) and (s, e) are not part of any subgame perfect equilibrium as deviating from e to s increases one's payoff by c and has no effect on the probability of success and so can not change expected continuation play by one's partner. However, the profiles may be realized in mixed strategy equilibria.

[5] This was first noted for infinitely repeated prisoner's dilemmas (Dal Bó and Fréchette (2011)), but has recently been applied to finitely repeated prisoner's dilemmas too (Embrey, Fréchette and Yuksel (2017)).

$\rho_T \geq \bar{\rho}_T = \frac{c}{(p-q)z}$ and i's expected payoff is given by the value function

$$V^T(\rho_T) = \max\{r_{\rho_T} z - c, qz\}$$

The optimal actions and values for earlier periods can be solved by backwards induction. Each action defines an expected payoff in period t, denoted $V^{t,e}$ for effort and $V^{t,s}$ for shirk, and Baye's rule determines how beliefs evolve. Formally, the period t belief ρ_t is the state-variable, and the dynamic programming problem to find the optimal actions and values is

$$\begin{aligned} V^{t,e}(\rho_t) &= r_{\rho_t} z - c + r_{\rho_t} V^{t+1}(\rho_{t+1}^{succ}) + (1 - r_{\rho_t}) V^{t+1}(\rho_{t+1}^{fail}) \\ V^{t,s}(\rho_t) &= qz + V^{t+1}(\rho_t) \\ V^t(\rho_t) &= \max\{V^{t,e}(\rho_t), V^{t,s}(\rho_t)\} \end{aligned}$$

where by Baye's Rule $$\rho_{t+1}^{succ} = \frac{\rho_t p}{r_{\rho_t}} \text{ and } \rho_{t+1}^{fail} = \frac{\rho_t(1-p)}{1 - r_{\rho_t}}$$

The following proposition shows that there is a cutoff belief in each period such that effort is optimal as long as beliefs put sufficient weight on effort by one's opponent.

Proposition 1. *There exists $\bar{\rho}_1 < \bar{\rho}_2 < \cdots < \bar{\rho}_T$ such that the optimal action in period t is e if and only if $\rho_t \geq \bar{\rho}_t$.*

The cutoffs are increasing, and so effort is a best response to more beliefs in the earlier periods of the game. This is because there are more periods left earlier in the game and so the information about whether one's partner is exerting effort or shirking that is only obtained with effort is more valuable. From the proposition, it is immediate that shirking in period t will lead to shirking in period $t+1$ (and all further periods) as beliefs will not change and the cutoff for effort to be a best response is only getting larger. On the other hand, while ρ increases after effort and success, the cutoffs also increase so the proposition leaves open the possibility that one might shirk after effort and success. However, the following theorem shows that this is not the case. First, I formally define a trigger strategy.

Definition 1. *A trigger strategy for player i is any strategy σ_i where*

- *For all h_i^t such that $a_{i,t'} = e$ for all $t' < t$ and $o_{i,t-1} = succ$, $\sigma_i(h_i^t) = e$.*
- *For all h_i^t such that $a_{i,t'} = s$ for some $t' < t$, $\sigma_i(h_i^t) = s$.*

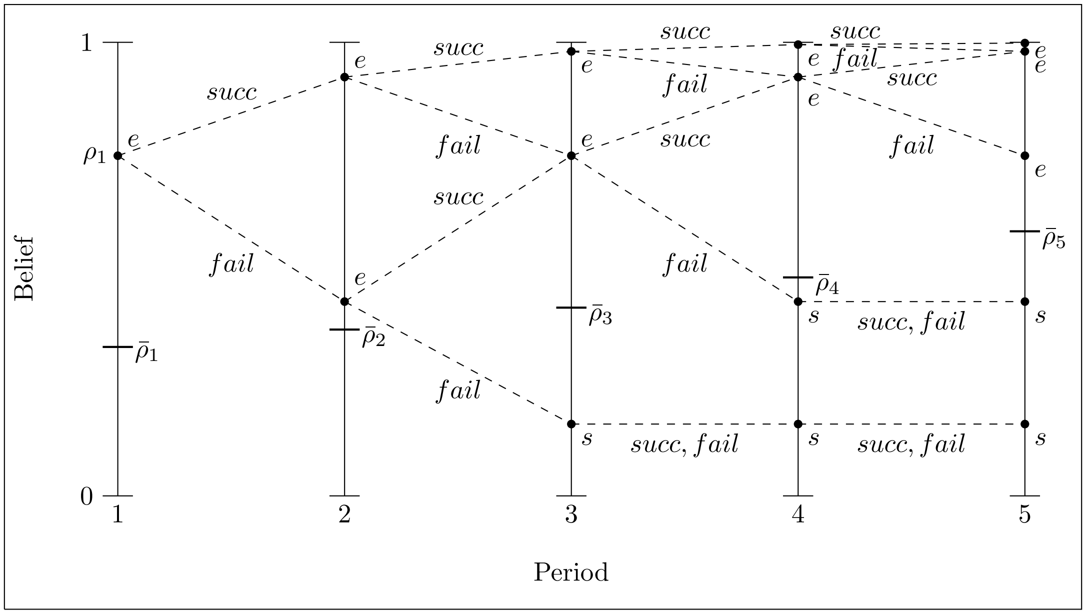


Figure 1: A Trigger Strategy Corresponding to Belief ρ_1

Theorem 1. *Suppose player i believes j plays always effort with probability ρ_1 and always shirk with probability $1 - \rho_1$. Then i's best response is a trigger strategy.*

Figure 1 illustrates the strategy for one particular initial belief ρ_1 in a $T = 5$ game using the parameters in the experiment (that will be stated later). Starting from the belief ρ_1, the figure illustrates how beliefs evolve after success or failure (the dashed lines) and how shirking is triggered when the belief falls below the cutoff found in Proposition 1. For this particular belief, shirking is permanently triggered in period 3 if both previous projects failed or in period 4 if two of the three previous projects failed.

2.3 Comparative Statics as T Increases

The motivation for this paper is to investigate an environment where repetition may decrease efficiency. The trigger strategies outlined in the previous section suggest this could be possible, because one shirk triggers permanent shirking so if the game is repeated enough times, then there could be significant shirking in later periods. So perhaps there is effort in the non-repeated game, but initial effort that gives way to permanent shirking in the repeated game. This is opposed to previously studied games, for instance the prisoner's dilemma, where efficiency is not possible in the non-repeated game and so repetition can only increase efficiency.[6]

However, there are two mitigating factors both stemming from the fact that the cutoffs $\bar{\rho}_t$ are increasing in t. Consider two repeated games with T and T' periods respectively where $T < T'$.[7] The fact that the value functions above are solved backwards means that the cutoffs are just a function of the number of periods remaining. This means that $\bar{\rho}_t$ is strictly decreasing in T for each period t. Therefore, more beliefs support effort in period 1 in the game with T' periods. And, for any fixed initial belief and history, a player who exerts effort in the game with T periods will also exert effort in the game with T' periods. That is, players wait longer to trigger shirking in longer games. The following theorem summarizes.

Theorem 2. *Fix $T < T'$.*

1. *$\bar{\rho}_1$ in the game with T repetitions is strictly larger than $\bar{\rho}_1$ in the game with T' repetitions.*

2. *Let σ_{i,ρ_1} denote i's optimal trigger strategy given period 1 beliefs ρ_1 from Theorem 1. Fix any $h_{i,t}$ with $t \leq T$. If $\sigma_{i,\rho_1}(h_{i,t}) = e$ in the game with T repetitions, then $\sigma_{i,\rho_1}(h_{i,t}) = e$ in the game with T' repetitions.*

To summarize, the probability of shirking only increases as the game progresses because trigger strategies permanently switch the optimal action to shirk. But on the other hand, more beliefs support initial effort and continued effort for games with more repetitions.

I conclude this section with a remark about why I use a finite horizon rather than an infinite horizon with discounting. The latter environment is relevant for the prisoner's dilemma as theory only predicts cooperation when the game is repeated infinitely. Here, there are multiple equilibria of the stage game so there are multiple equilibria of the finitely repeated game too and so infinite repetition is unnecessary. Furthermore, given that it is impossible to truly run an infinite horizon game in the laboratory, I think that the finite horizon makes more sense. However, the model of strategic uncertainty would make very similar predictions in the infinite horizon with the discount factor providing the comparable comparative statics to the number of repetitions that I use here.

[6] Formally, defection (the inefficient action) is a dominant strategy in the prisoner's dilemma so, even in the model of strategic uncertainty, it is always the best response in the non-repeated game.

[7] So far, this has been framed as comparing the repeated and non-repeated game although the experiment considers $T = 1$ (the non-repeated game) and three case with $T > 1$ (repeated games) so from now on I will discuss the number of periods T.

3 Experimental Design

3.1 Treatments

The parameters of the stage game were fixed in all treatments at the following values: $p = .8$, $q = .2$, $z = 100$, and $c = 35$. Players also received 50 each period so no payoffs would be negative leading to the game matrix

	e	s
e	(95,95)	(35,70)
s	(70,35)	(70,70)

The parameters were chosen so that (s, s) is risk dominant but not by much (in the sense that the expected payoff of a player who thinks effort and shirk are equally likely by their partner is 65 for effort and 70 for shirk which are not too different). This calibration was intended to obtain significant, but less than 100%, effort in the non-repeated game. Significant effort provides a large baseline against which effort in repeated games can decrease. While 100% effort would create the largest baseline, it would also probably mean beliefs very close to $\rho_1 = 1$ which would translate to near 100% effort in repeated games too. Hence, significantnt, but not excessive, effort in the non-repeated game provides the best-case scenario for decreasing efficiency with repetition.

The treatment variable was the number of repetitions T which took on the values 1, 5, 10, and 25. For convenience, these treatments are named $T = 1$, $T = 5$, $T = 10$, and $T = 25$ respectively.

A total of 194 undergraduate students at the University of Virginia were recruited through the Veconlab software to participate. There were 4 sessions of each of the 4 treatments and the number of subjects per treatment was relatively balanced (46, 52, 46, and 50 respectively). No student participated in more than 1 session. I call each play of the game a match. Sessions consisted of 20 matches for Treatments $T = 1$, $T = 5$, and $T = 10$ and 10 matches for Treatment $T = 25$.[8] Subjects were randomly matched into pairs for each match.

At the end of each period, subjects observed whether the project succeeded or failed. This information, along with their own action, remained on their screen for the rest of the experiment. They never saw their partner's actions.

[8] In Treatment $T = 1$, subjects also played more games after the 20 matches. They did not see the instructions for these games until they finished the 20 matches although they were told there would be a second part to the experiment that was independent of the first part. The data from these extra games is not relevant for this paper.

All terms were neutrally framed and the experiment was programmed and run in z-tree (Fischbacher 2007). Instructions for Treatment $T = 5$ are in Appendix B and instructions for other treatments were similar. Subjects earned points in each match, but were paid for just 1 match that was randomly selected and revealed to them at the end of the experiment. The points were converted into dollars at the rate of 20, 20, 30, and 75 points per dollar for the four respective treatments.[9] They also got $6 for participating. Average earnings were $30.39.

3.2 Predictions

The predictions for behavior are based on the model of strategic uncertainty developed in Section 2.2 and predictions for treatment effects come from Section 2.3. The main prediction for behavior is that subjects will use trigger strategies. An easy way to test for trigger strategies is to consider the effort rates (i.e. the proportion of choices that are effort) in period t (where $t > 1$) conditional on the choice and outcome in period $t - 1$: trigger strategies predict effort in period t after effort and success in period $t-1$, effort or shirk in period t after effort and failure in period $t-1$, and shirk in period t after shirk and either success or failure in period $t - 1$. This translates to the following hypothesis.

Hypothesis 1. *Subjects will use trigger strategies. Effort rates in period* t *conditional on the choice and outcome in period* $t - 1$ *will be ranked*

$$1 = (e, succ) > (e, fail) > (s, succ) = (s, fail) = 0$$

Theorem 2 provides predictions for treatment effects. The theorem concerns beliefs, so I further assume that the more beliefs that support effort, the more effort that subjects will exert. This is a standard assumption in the literature on repeated prisoner's dilemmas (see Dal Bó and Fréchette (2011)).

The first point in Theorem 2 forms a testable hypothesis for effort in period 1. The theorem states that more beliefs will support effort when the stage game is repeated more times so effort rates are predicted to be increasing in the number of repetitions.

Hypothesis 2. *Period 1 effort rates will be ranked from most to least:*

$$T{=}25 \;>\; T{=}10 \;>\; T{=}5 \;>\; T{=}1$$

[9] They also earned money in the second part of the experiment for the $T = 1$ treatment.

The second point in Theorem 2 is that optimal trigger strategies switch to shirking only after more failures when the game is repeated more times. While I will analyze this a couple of ways in the results, one straightforward and testable prediction of this result is that effort rates will decrease (because trigger strategies are employed), but at a smaller rate, as the number of repetitions increases.

Hypothesis 3. *Effort rates will decrease as the games progress. The rates of decrease will be ranked from most to least:*

$$T{=}5 \; > T{=}10 \; > \; T{=}25$$

Putting the three hypotheses together, it is the case that comparing total effort rates, and thus total efficiency, is ambiguous. As such, no prediction will be made for total effort rates, although it is perhaps the preeminent question posed in the experiment and of course will be addressed in the results.

4 Results

In order to incorporate experience all the following results consider the last half of matches (matches 11-20 in Treatments $T = 1$, $T = 5$, and $T = 10$ and matches 6-10 in Treatment $T = 25$).[10] All statistical results use non-parametric tests with data averaged at the session level.

Hypothesis 1 states that subjects will use trigger strategies. The first way I analyze this is to simply look at the proportion of matches where the choices are exactly consistent with play of a trigger strategy. This proportion is .83 for $T = 5$, .80 for $T = 10$, and .74 for $T = 25$.

These proportions are quite large which indicates pretty strong evidence for trigger strategies. Hypothesis 1 makes a precise prediction for effort conditional on the previous period choice and outcome. Figure 2 disaggregates the effort rates (for periods greater than one) in this way to illustrate the result.

The figure indicates strong support for the hypothesis. There is only about 10% effort after a player shirked in the previous period. Furthermore, there is nearly 100% effort after a player exerted effort and the project succeeded in the previous period. Statistically, pairwise comparisons yield that there is more effort after $(e, succ)$ than $(e, fail)$ ($p < .05$ for each treatment, Wilcoxon-Mann-Whitney test), more effort

[10] There are actually very few differences if all matches are considered. Learning appeared limited in this paper, perhaps because subjects never received feedback on the choice of their opponent.

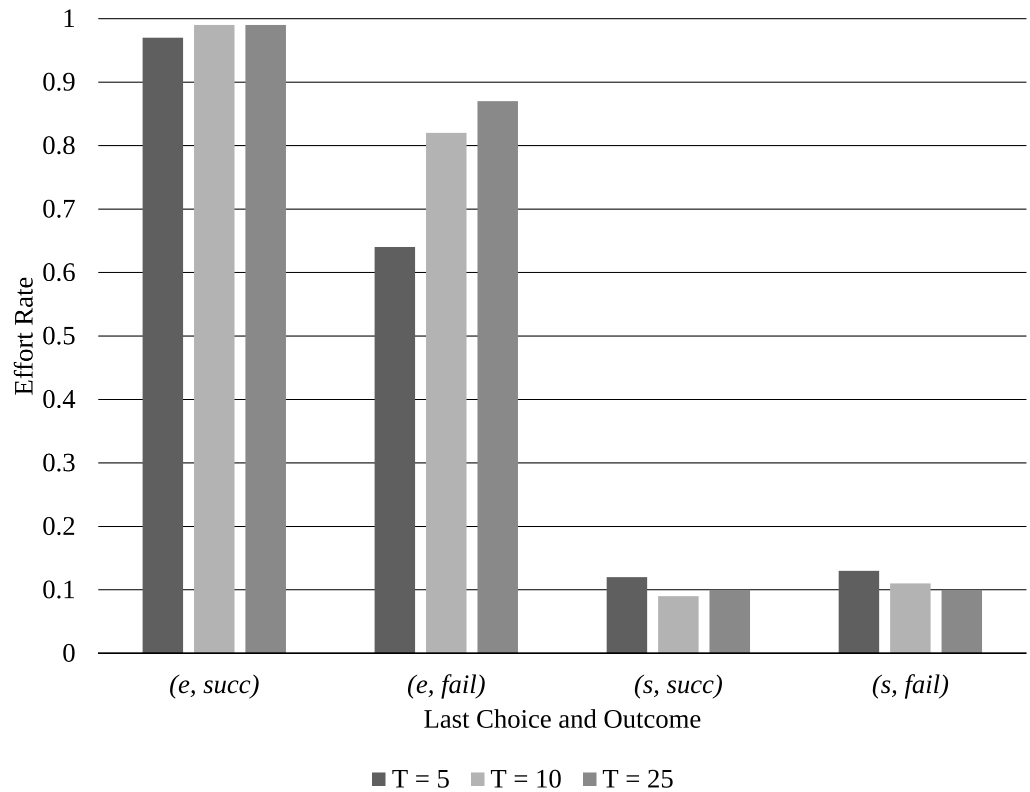


Figure 2: Effort Rates By Last Choice and Outcome: $t > 1$

after $(e, fail)$ than either $(s, succ)$ or $(s, fail)$ ($p < .05$ for each treatment for each test, Wilcoxon-Mann-Whitney test), and no difference between $(s, succ)$ and $(s, fail)$ ($p > .1$ for each treatment, Wilcoxon-Mann-Whitney test). Trigger strategies seem to be ubiquitous and the first result summarizes.

Result 1. *The large majority of play is consistent with trigger strategies.*

I now move to comparing behavior between treatments. Figure 3 disaggregates the effort rates by period to illustrate the main results.

The prediction of Hypothesis 2 is that there will be a positive relationship between period 1 effort and T. The data points corresponding to period 1 in the figure indicate that this is indeed true. Specifically, the period 1 effort rates are .66 in Treatment $T = 1$, .74 in Treatment $T = 5$, .80 in Treatment $T = 10$, and .87 in Treatment $T = 25$. Statistically, the positive trend is significant ($p < .05$, one-tail Jonckheere-Terpstra test).

The prediction of Hypothesis 3 is that effort rates will decrease as the game progresses, but at a smaller rate when T is larger. Figure 3 indicates that this is exactly

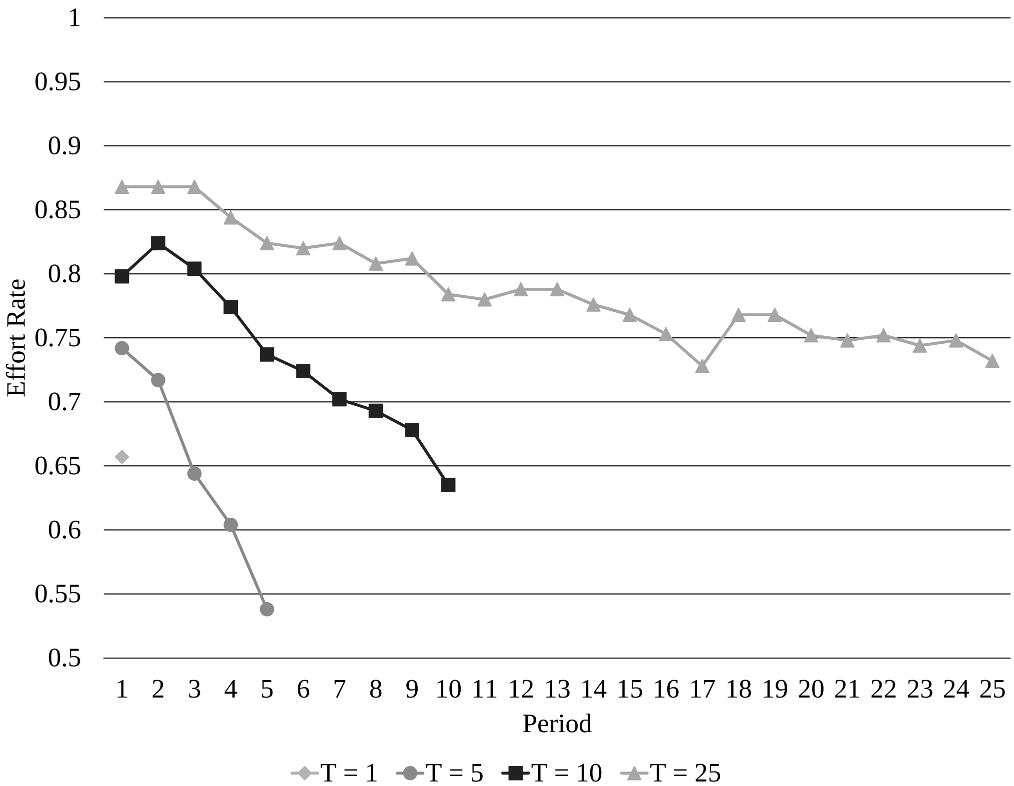


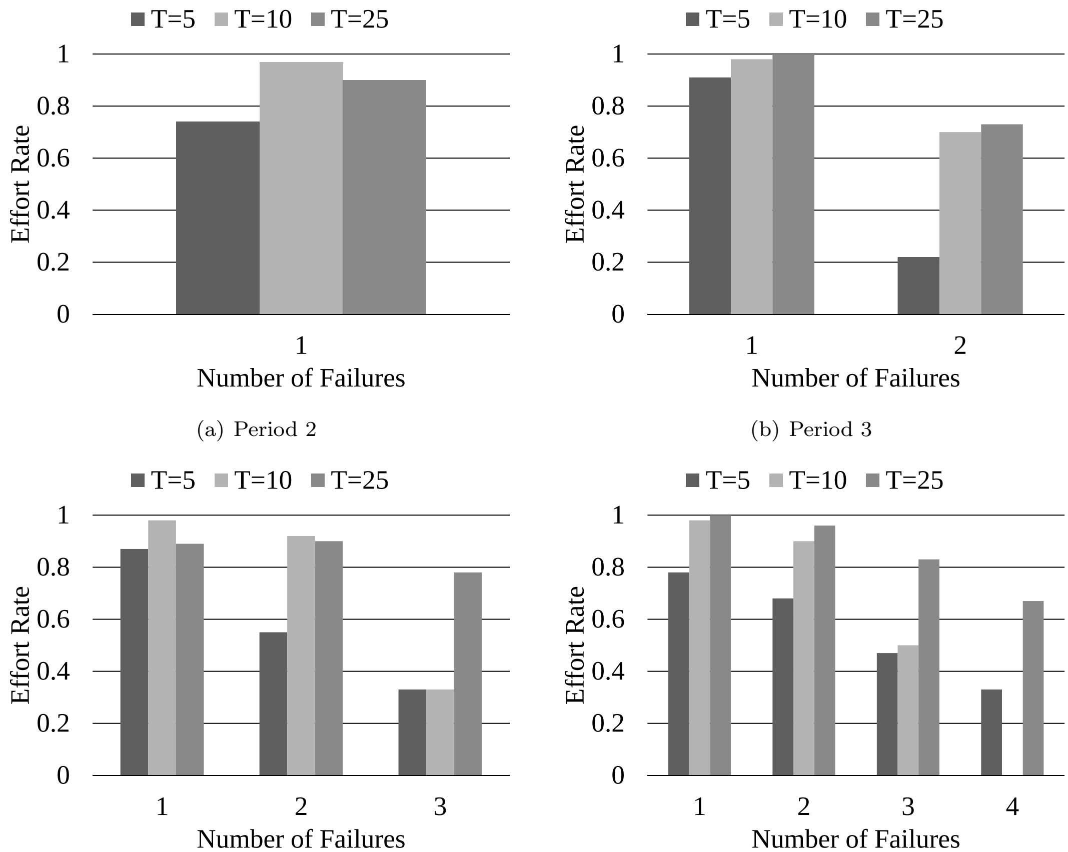
Figure 3: Effort Rates By Period

what happens. The average decrease per period is 4 percentage points for Treatment $T = 5$, 1.6 percentage points for Treatment $T = 10$, and .5 percentage points for Treatment $T = 25$. These decreases are significantly negative for 2 of the 3 treatments ($p < .1$ for $T = 5$ and $T = 10$, Wilcoxon signed-rank test).[11] Furthermore, they are less negative as T increases ($p < .01$, one-tail Jonckheere-Terpstra test).

There is also evidence in support of Hypothesis 3 in Figure 2. The intermediate effort rates after $(e, fail)$ are increasing in T as slower triggers would predict. The rates are .64 in Treatment $T = 5$, .81 in Treatment $T = 10$, and .87 in Treatment $T = 25$. The positive trend is significant ($p < .01$, one-tail Jonckheere-Terpstra test). This is the history where trigger strategies can switch to shirking and, indeed, the switch is less common for games with more repetitions.

Figure 4 delves deeper into the trigger strategies to provide more precise evidence

[11] These are tests with 4 sessions each so the best possible significance level is just $p < .1$ (when all 4 sessions are negative). In total, the decrease was negative in 11/12 sessions. The one session (in $T = 25$) where it increased was from effort in 67/70 period 1 choices to effort in 68/70 period 25 choices so this session was essentially just a session where everyone chose the always effort trigger strategy.



(a) Period 2
(b) Period 3
(c) Period 4
(d) Period 5

Figure 4: Effort Rates By Number of Fails and Period: $a_{i,t'} = e$ for all $t' < t$ and $o_{t-1} = fail$

that switches to shirking require more failures as the number of repetitions increase. This figure illustrates effort rates in period t conditional on the period and number of fails (as well as effort has only been exerted so far and that the outcome in period $t-1$ was failure).[12]

Broadly speaking, the figure indicates that shirking is triggered after fewer fails when the game is shorter which is direct evidence in support of Theorem 2. I highlight the main findings for each period.

In period 2, failure in the first period sometimes triggers shirking in Treatment $T = 5$ (26% of the time) but rarely does in Treatments $T = 10$ or $T = 25$ (less than 10% of the time). One of the two comparisons is significant ($p < .05$ for comparison of $T = 5$ to $T = 10$, Wilcoxon-Mann-Whitney test).

[12] The number of fails is a sufficient statistic (i.e. the full sequence of previous outcomes is unnecessary), because $q = 1 - p$ so every increase in beliefs from a success is exactly canceled out by the decrease from a failure. This point is illustrated in Figure 1.

In period 3, failures in both previous periods usually triggers shirking in Treatment $T = 5$ (78% of the time) but does not usually trigger shirking in Treatments $T = 10$ or $T = 25$ (less than 30% of the time). Both comparisons are significant ($p < .05$ for both comparisons, Wilcoxon-Mann-Whitney test).

In period 4, 2 previous failures triggers shirking about half the time in Treatment $T = 5$ (45% of the time) but rarely does in Treatments $T = 10$ or $T = 25$ (less than 10% of the time). Both comparisons are at least marginally significant ($p < .1$ for comparison of $T = 5$ to $T = 10$ and $p < .05$ for comparison of $T = 5$ to $T = 25$, Wilcoxon-Mann-Whitney test). Also, the first difference between the longer treatments appears here where failures in all three previous periods trigger shirking most of the time in Treatments $T = 5$ and $T = 10$ (67% of the time in both) but not that often in Treatment $T = 25$ (22% of the time). Both comparisons are at least marginally significant ($p < .05$ for comparison of $T = 5$ to $T = 25$ and $p < .1$ for comparison of $T = 10$ to $T = 25$, Wilcoxon-Mann-Whitney test).

In period 5, there is definitely less effort in Treatment $T = 5$, but there also seems to be an endgame effect here (there is decreased effort even after just 1 failure and 3 successes) so this could be helping obtain the result. There are still differences between the longer treatments where 3 or 4 previous fails trigger shirking often in Treatment $T = 10$ (50% and 100% of the time respectively) but still do not often trigger fails in Treatment $T = 25$ (17% and 33% of the time respectively). One of the two comparisons is marginally significant ($p < .1$ for 4 fails, Wilcoxon-Mann-Whitney test).

Overall, the evidence indicates strong support for both Hypotheses 2 and 3. The second result summarizes.

Result 2. *There is more period 1 effort when T is larger. Effort rates decrease across time in all repeated games, but shirking is triggered less often when T is larger.*

Finally, I can investigate perhaps the preeminent question. Is there a relationship between total effort (i.e. efficiency) and T? Effort rates are higher at the beginning of the game when T is larger, but they also only drop across time, although at a lower rate when T is larger. So the answer is not obvious. The following table provides the result.

Total Effort

Treatment	Effort Rate
OS	.66
5 Per	.65
10 Per	.74
25 Per	.79

There appears to be a slightly positive trend, although this trend is not exactly monotonic and it is not significant ($p > .1$, two-tail Jonckheere-Terpstra test).[13]

Result 3. *Total effort and efficiency do not decrease as the number of repetition increase. There is perhaps a little more total effort as the number of repetitions increase.*

5 Conclusion

In this paper, I evaluate results from a laboratory experiment on a repeated partnership game with imperfect public monitoring. Subject exert more effort in period 1 when the number of repetitions is larger, and effort rates decrease as periods progress but at a slower rate for games with more repetitions. These results are consistent with a theory of strategic uncertainty in which a subject is uncertain as to whether their partner exerts effort or shirks, and thus the outcomes of projects play a significant signaling role for determining the best response.

In total, there is not less effort and more inefficiency when there are more repetitions. This leaves open the question of whether repetition could ever decrease efficiency. While other games perhaps would be more amenable, the parameters in this experiment seem to give decreased efficiency the best shot within the class of partnership games for several reasons. First, the effort rates in the non-repeated game are right around 2/3 which allows room for repetition to decrease effort, but is not also so large as to think subjects would put near-one weight on their partner choosing effort in the first period of the repeated game which would lead them to ignore all failures and just keep exerting effort. Second, the information structure is designed so that shirking is an absorbing state and so relationships that go bad can never recover.

[13]It is significant at the 10% level for a one-tail test, but as there is no a priori hypothesis about the direction of the trend the two-tail test is appropriate.

Appendix A: Proofs

Proof of Proposition 1:

That $\bar{\rho}_T = \frac{c}{(p-q)z}$ is immediate given that effort is optimal if and only if $r_{\rho_T} z - c \geq qz$. For the other statements, I actually prove all of the following:

- There exists $0 < \bar{\rho}_1 < \bar{\rho}_2 < \cdots < \bar{\rho}_T$ such that the optimal action in period t is e if and only if $\rho_t \geq \bar{\rho}_t$.
- For each period t, $V^t(\rho_t)$ is constant for all $\rho_t \leq \bar{\rho}_t$.
- For each period t, $V^t(\rho_t)$ is strictly increasing for all $\rho_t \geq \bar{\rho}_t$.
- For each period t, $V^t(\rho_t)$ is continuous and weakly increasing for all ρ_t.

I prove the four statements simultaneously by backwards induction starting from period T:

Base Case: period T:

The existence of $\bar{\rho}_T$ is established. If $\rho_T \leq \bar{\rho}_T$, $V^T(\rho_T) = qz$ which does not depend on ρ_T. If $\rho_T > \bar{\rho}_T$, $V^T(\rho_T) = r_{\rho_T} z - c$ which is strictly increasing in ρ_T. The two functions are clearly continuous and both approach qz as ρ approaches $\bar{\rho}_T$ so it is continuous at $\bar{\rho}_T$ too and weakly increasing everywhere.

Inductive Hypothesis (IH): Suppose the statements hold for period $t+1$:

First, $r_{\rho_t} z - c + r_{\rho_t} V^{t+1}(\rho_{t+1}^{succ}) + (1 - r_{\rho_t}) V^{t+1}(\rho_{t+1}^{fail})$ and $qz + V^{t+1}(\rho_t)$ are each continuous because they add a linear function to value functions that are continuous by the IH. So it is immediate that $V^t(\rho_t)$ is continuous because it is the maximum of these two functions.

Now, I show the existence of the cutoff $\bar{\rho}_t$ such that effort is optimal if and only if $\rho_t \geq \bar{\rho}_t$. By construction effort is optimal iff

$$r_{\rho_t} z - c + r_{\rho_t} V^{t+1}(\rho_{t+1}^{succ}) + (1 - r_{\rho_t}) V^{t+1}(\rho_{t+1}^{fail}) \geq qz + V^{t+1}(\rho_t)$$

The left hand side is strictly increasing in ρ_t, which can be seen by noting that

- r_{ρ_t} is strictly increasing in ρ_t
- $V^{t+1}(\rho_{t+1}^{succ})$ and $V^{t+1}(\rho_{t+1}^{fail})$ are weakly increasing in ρ_t by the IH.
- $V^{t+1}(\rho_{t+1}^{succ}) \geq V^{t+1}(\rho_{t+1}^{fail})$, because $\rho_{t+1}^{succ} \geq \rho_{t+1}^{fail}$ by Bayes rule and the function is weakly increasing by the IH.

- Combining the two previous points, it is the case that more weight (r_{ρ_t} is increasing) is put on the larger value ($V^{t+1}(\rho_{t+1}^{succ})$) as ρ_t increases.

The right hand side is constant in ρ_t for all $\rho_t \leq \bar{\rho}_{t+1}$ by the IH. So I consider two cases:

Case 1: $\rho_t \leq \bar{\rho}_{t+1}$

Shirk is optimal for $\rho_t = 0$ because then $\rho_{t+1}^{succ} = \rho_{t+1}^{fail} = \rho_t = 0$ by Baye's rule so the inequality reduces to $qz - c < qz$. As the left hand side is increasing in ρ_t and the right hand side is constant there is at most 1 cutoff such that effort is optimal if and only if ρ_t exceeds this cutoff.

Case 2: $\rho_t \geq \bar{\rho}_{t+1}$

By the IH, effort is optimal in period $t+1$ if the belief is ρ_t. After shirking in period t this is exactly the case so the right hand side of the equation is

$$qz + r_{\rho_t}z - c + r_{\rho_t}V^{t+2}(\rho_{t+1}^{succ}) + (1 - r_{\rho_t})V^{t+2}(\rho_{t+1}^{fail})$$

On the other hand, by construction $V^{t+1}(\rho) \geq qz + V^{t+2}(\rho)$ for all ρ. Hence the left hand side of the inequality is at least

$$r_{\rho_t}z - c + qz + r_{\rho_t}V^{t+2}(\rho_{t+1}^{succ}) + (1 - r_{\rho_t})V^{t+1}(\rho_{t+2}^{fail})$$

So the left hand side is weakly larger and thus effort is optimal for all $\rho_t \geq \bar{\rho}_{t+1}$.

Combining the two cases, and because $V^t(\rho_t)$ is continuous, there exists $\bar{\rho}_t$ such that effort is optimal if and only if $\rho_t \geq \bar{\rho}_t$. What remains is to show that $\bar{\rho}_t < \bar{\rho}_{t+1}$.

Towards this goal, suppose $\rho_t = \bar{\rho}_{t+1}$. I first show effort is strictly better than shirk in this case. That is,

$$r_{\bar{\rho}_{t+1}}z - c + r_{\bar{\rho}_{t+1}}V^{t+1}(\rho_{t+1}^{succ}) + (1 - r_{\bar{\rho}_{t+1}})V^{t+1}(\rho_{t+1}^{fail}) > qz + V^{t+1}(\bar{\rho}_{t+1}) \quad (1)$$

By Baye's rule we have $\rho_{t+1}^{succ} > \rho_t = \bar{\rho}_{t+1} > \rho_{t+1}^{fail}$ ($\bar{\rho}_{t+1} > 0$ by the IH so the inequalities are indeed strict). By the IH, this means that $V^{t+1}(\rho_{t+1}^{succ}) > V^{t+1}(\bar{\rho}_{t+1}) = V^{t+1}(\rho_{t+1}^{fail})$ so Equation (1) simplifies to

$$r_{\bar{\rho}_{t+1}}z - c + r_{\bar{\rho}_{t+1}}[V^{t+1}(\rho_{t+1}^{succ}) - V^{t+1}(\rho_{t+1}^{fail})] > qz \quad (2)$$

By construction, we always have $V^{t+1}(\rho) \geq qz + V^{t+2}(\rho)$ for all ρ. By the IH, this inequality is strict if effort is the strict best response in period $t+1$. By construction, it is an equality if shirk is the best response. Hence $V^{t+1}(\rho_{t+1}^{succ}) > qz + V^{t+2}(\rho_{t+1}^{succ})$ and

$V^{t+1}(\rho_{t+1}^{fail}) = qz + V^{t+2}(\rho_{t+1}^{fail})$. Together, these imply that $V^{t+1}(\rho_{t+1}^{succ}) - V^{t+1}(\rho_{t+1}^{fail}) > V^{t+2}(\rho_{t+1}^{succ}) - V^{t+2}(\rho_{t+1}^{fail})$. Hence, it is true that

$$r_{\bar{\rho}_{t+1}}z - c + r_{\bar{\rho}_{t+1}}[V^{t+1}(\rho_{t+1}^{succ}) - V^{t+1}(\rho_{t+1}^{fail})] > r_{\bar{\rho}_{t+1}}z - c + r_{\bar{\rho}_{t+1}}[V^{t+2}(\rho_{t+1}^{succ}) - V^{t+2}(\rho_{t+1}^{fail})] \quad (3)$$

By choosing $\rho_t = \bar{\rho}_{t+1}$, it is also true that effort and shirk give equal value in period $t+1$. That is,

$$r_{\bar{\rho}_{t+1}}z - c + r_{\bar{\rho}_{t+1}}V^{t+2}(\rho_{t+1}^{succ}) + (1 - r_{\bar{\rho}_{t+1}})V^{t+2}(\rho_{t+1}^{fail}) = qz + V^{t+2}(\bar{\rho}_{t+1}) \quad (4)$$

Just as Equation (1) simplified to Equation (2), the same is true here (using the IH hypothesis from the step $t+2$) so Equation (4) simplifies to

$$r_{\bar{\rho}_{t+1}}z - c + r_{\bar{\rho}_{t+1}}[V^{t+2}(\rho_{t+1}^{succ}) - V^{t+2}(\rho_{t+1}^{fail})] = qz \quad (5)$$

Combining Equations (3) and (5) implies Equation (2) as desired.

As the left hand side of Equation (2) is strictly increasing in ρ_t and the right hand side is constant (as established earlier in the proof), the value that solves it at equality is strictly less than $\bar{\rho}_{t+1}$.

Finally, $V^t(\rho_t)$ is strictly increasing for $\rho_t \geq \bar{\rho}_t$ as it is the left hand side of Equation (2) and constant for $\rho_t \leq \bar{\rho}_t$ as it is the right hand side. Combining this with continuity implies it is weakly increasing for all ρ_t. □

Proof of Theorem 1:

Suppose $h_{i,t}$ is history where $a_{i,t'} = s$ for some $t < t'$. Then $\rho_{t'+1} = \rho_{t'}$ and so $\rho_{t'+1} \leq \bar{\rho}_{t'} < \bar{\rho}_{t'+1}$ for the cutoff $\bar{\rho}_{t'+1}$ defined in Proposition 1. Hence shirk is optimal in period $t'+1$. The same goes for periods $t'+2$ up to t.

Suppose $h_{i,t}$ is a history where $a_{i,t'} = e$ for all $t' < t$ and $o_{i,t-1} = succ$. Suppose for contradiction that $\sigma(h_{i,t}) = s$. This means that for period $t-1$ belief ρ_{t-1} it is the case that $\rho_{t-1}^{succ} \leq \bar{\rho}_t$. Then, in period $t-1$

$$r_{\rho_{t-1}}z - c + r_{\rho_{t-1}}V^t(\rho_{t-1}^{succ}) + (1 - r_{\rho_{t-1}})V^t(\rho_{t-1}^{fail}) \geq qz + V^t(\rho_{t-1})$$

By Baye's rule we have $\rho_{t-1}^{fail} \leq \rho_{t-1} \leq \rho_{t-1}^{succ} \leq \bar{\rho}_t$. By Proposition 1, $V^t(\rho_{t-1}^{fail}) = V^t(\rho_{t-1} \leq \rho_{t-1}^{succ}) = V^t(\bar{\rho}_t)$ so the inequality reduces to $r_{\rho_{t-1}}z - c \geq qz$ when means that $\rho_{t-1} \geq \bar{\rho}_T$. This is a contradiction because $\bar{\rho}_t < \bar{\rho}_T$ so we cannot have $\rho_{t-1} \leq \bar{\rho}_t$

and $\rho_{t-1} \geq \bar{\rho}_T$. $\square$

Proof of Theorem 2:

Rewrite Proposition 1 as there exists $\bar{\rho}^T < \bar{\rho}^{T-1} < \cdots < \bar{\rho}^1$ such that effort is optimal iff $\rho_t \geq \bar{\rho}^{T-t+1}$. That is $\bar{\rho}^t$ is the cutoff when there are t periods remaining. As the proof was by backwards induction, the proof of this rewritten proposition is identical and $\bar{\rho}_t = \bar{\rho}^{T-t+1}$. Now the two statements are almost immediate.

1. The first statement follows because $\bar{\rho}^T = \bar{\rho}_1$ in the game with T periods and $\bar{\rho}^{T'} = \bar{\rho}_1$ in the game with T' periods and $\bar{\rho}^{T'} < \bar{\rho}^T$.

2. The second statement follows because at history $h_{i,t}$ where $\sigma_{i,\rho_1}(h_{i,t}) = e$ in the game with T repetitions it must be that $\rho'_t \geq \bar{\rho}_{t'}$ for all $t' < t$. By the rewritten Proposition, $\bar{\rho}^{T-t'+1} = \bar{\rho}_{t'}$ in the game with T periods and $\bar{\rho}^{T'-t'+1} = \bar{\rho}_{t'}$ in the game with T' periods and $\bar{\rho}^{T'-t'+1} < \bar{\rho}^{T-t'+1}$ for all $t' \leq t$. Hence, effort is optimal and beliefs evolve identically in all periods $t' \leq t$ in the game with T' repetitions.

Appendix B: Instructions

Welcome. This is an experiment in decision making. Various research foundations and institutions have provided funding for this experiment and you will have the opportunity to make a considerable amount of money which will be paid to you at the end. Make sure you pay close attention to the instructions because the choices you make will influence the amount of money you will take home with you today. Please ask questions if any instructions are unclear.

The Choice Problem

In this experiment, you will engage in the following two-person choice problem with one other participant from this room. There is a project with two possible outcomes: High or Low. The outcome is determined by the choices of you, the other participant, and chance in the following way. You and the other participant will each simultaneously choose between two options which we have labeled A and B.

- If you both choose B: then the computer randomly draws a number between 0 and 100 and if the number is at least 20 the outcome is High and if the number is less than 20 the outcome is Low. In other words, if you both choose B, there is an 80% chance the project outcome will be High.
- If either, or both, of you choose A: then the computer randomly draws a number between 0 and 100 and if the number is at least 80 the outcome is High and if the number is less than 80 the outcome is Low. In other words, if you do not both choose B, there is a 20% chance the project outcome will be High.

If the project outcome is High, both you and the other participant receive 100 points. If it is Low, then you both receive 0 points. Each person also receives 50 points if they choose A and 15 points if they choose B.

The following table summarizes your expected payoff which is the number of points you can expect on average for each combination of choices by you and the other participant:

Your Choice	Others Choice	Points for Your Choice	High Chance	Your Expected Payoff	Others Expected Payoff
A	A	50	20%	50+100(20%)=70	50+100(20%)=70
A	B	50	20%	50+100(20%)=70	15+100(20%)=35
B	A	15	20%	15+100(20%)=35	50+100(20%)=70
B	B	15	80%	15+100(80%)=95	15+100(80%)=95

One final way to say this is that, if you choose A, then you get 70 points on average regardless of what the other participant chooses and, if you choose B, the you get 95 points on average if the other participant also chooses B and 35 points on average if they choose A.

Procedures

The experiment will consist of 20 of the following Matches. At the beginning of each Match you will be randomly matched with another participant. You will then engage in the choice problem described above 5 times, which we will call Rounds, with this same other participant. After the 5 Rounds of the Match are completed, you will then be randomly matched with a different participant to start the next Match. In other words, you will engage in the choice problem with the same other participant for all 5 Rounds of each Match, but then get randomly matched with a new participant between Matches.

At the end of every Round, you will be told whether the outcome of the project was High or Low and then receive the points corresponding to your choice and the outcome of the project. This information will remain on your screen as the experiment progresses. You will not be told what choice the other participant made.

At the end of the experiment, the computer will randomly select one Match for payment. We will give you 1 dollar for every 20 points you earned in the 5 Rounds of the selected Match. You will also get 6 dollars for participating.

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