PRICE DISCRIMINATION
Two passengers traveling on the same flight in economy class may have paid different amounts.
It appears that they both purchased the same good, however
• one ticket may have restrictions
• one may have purchased the ticket with more anticipation
Price discrimination is the practice of setting different prices for (approximately) the same good.
A necessary condition for price discrimination is the absence of resale:

- For example, since electricity can not be stored, no resale can take place.
- Software licences are not transferable, thus they can not be sold in a secondary market.
PERFECT PRICE DISCRIMINATION
Consider the case where a total of $n$ consumers, have each a valuation $v_i$ for one unit of a given product.
Let us further assume that for all $i, j \in \{1, 2, \ldots, n\}$
\[ v_i < v_j \iff i < j \]
In the graphical example let us assume $n = 4$ and the monopolist’s with marginal cost $c < v_1$ sets a price $p^* = v_2$
The consumer surplus is:
\[ CS(p^*) = (v_3 - p^*) + (v_4 - p^*) \]
Inverse Demand

P(Q)

v_1

v_2

v_3

v_4

1 2 3 4

Q
If the valuations are known to the monopolist, he/she can extract all consumer surplus by charging different prices depending upon each consumer's valuations. This is known as “first degree” price discrimination.
Let us assume now that all \( n \) consumers are identical, i.e. for a consumption level \( q \), they all have a valuation of \( p(q) \). Aggregate demand is therefore

\[
D(p) = nq(p)
\]

Rather than setting a price

\[
p^m \in \arg \max_p [(p - c)D(p)]
\]
The monopolist can do better by setting a two part tariff \((A, p)\) where

\[
p = c
\]

\[
A = \frac{CS(c)}{n}
\]
OTHER TYPES OF PRICE DISCRIMINATION

When buyers characteristics are observable, the seller can establish different prices as a function of the buyer’s characteristic.

- Examples include membership discounts, student discounts for software, senior citizen discounts.
- This type of discrimination is referred to as “third-degree” price discrimination.
Two demand types $D_1(p)$ and $D_2(p)$ then monopolist solves

$$\max_{p_1, p_2} [p_1 D_1(p_1) + p_2 D_2(p_2) - C(D_1(p_1) + D_2(p_2))]$$

First order condition implies

$$MR_1 = MR_2 = MC$$
This leads to

\[
\frac{p_1^* - C'}{p_1^*} = \frac{1}{\epsilon_1} \quad \text{and} \quad \frac{p_2^* - C'}{p_2^*} = \frac{1}{\epsilon_2}
\]

optimal pricing implies that the monopolist should charge more in markets with the lower elasticity of demand.
EXAMPLE: PEAK-LOAD PRICING

A monopolist faces two types of demand: off-peak $D_1(p)$ and peak $D_2(p)$ where

$$D_1(p) < D_2(p)$$

Assuming linear marginal cost and a capacity $K$, optimal prices solve the following problem

$$\max_{p_1, p_2} (p_1 - c)D_1(p_1) + (p_2 - c)D_2(p_2)$$

s.t

$$D_1(p_1) \leq K$$

$$D_2(p_2) \leq K$$
Since only the second constraint can be binding, the Lagrangian is:

\[ \mathcal{L}(p_1, p_2, \mu) = (p_1 - c)D_1(p_1) + (p_2 - c)D_2(p_2) + \mu(K - D_2(p_2)) \]

where \( \mu \geq 0 \) is the shadow price (or Lagrange multiplier) associated with the binding constraint. The first order condition is:

\[ D_1(p_1) + (p_1 - c)D_1'(p_1) = 0 \]

\[ D_2(p_2) + (p_2 - c)D_2'(p_2) - \mu D_2'(p_2) = 0 \]
which leads to

\[
\frac{p_1^* - c}{p_1^*} = \frac{1}{\epsilon_1}
\]

\[
\frac{p_2^* - c - \mu}{p_2^*} = \frac{1}{\epsilon_2}
\]
NONLINEAR PRICES
When the seller can not observe the characteristics of each buyer and offers a menu of selling contracts.

- Customers self-select according to their own characteristics.
- Example, airfares that require buyer to stay over the weekend (non-business travelers self select this option)
- This is known as “second degree” price discrimination
PRICING TELECOM SERVICES

Two types of customers;

\[ CS_2(p) > CS_1(p) \]

If the seller could determine customer’s types, he/she would simply use two two-part tariffs with variable charge equal to marginal cost and fixed charge equal to customer’s surplus. Instead the seller offers the choice to customers (example, optional calling plans).
• If the seller wants customers to *self-select*, it must make sure that type 2 customers have no incentive to adopt the calling plan that is intended for type 1 customers. This is known as the *incentive compatibility* constraint.

• The seller also must make sure that each type of customer prefers to pay the fixed fee and consume its optimal quantity rather than not consume at all. This is known as the *individual rationality* constraint.
In an optimal pricing schedule,

• the high consumption type pays a high fixed fee but a low variable charge

• the low consumption type pays a low fixed fee but a higher variable charge
NONLINEAR PRICING

There are two types of buyers; ones with *high* valuation, i.e. if they are charged a price $p$ for an amount $q$ their utility is

$$U_1^H = v_H(q) - pq$$

There are also buyers with *low* valuation, i.e. if they are charged a price $p$ for an amount $q$ their utility is

$$U_1^L = v_L(q) - pq$$
where

\[ v_H(q) > v_L(q) \]

and \( v'_i(\cdot) > 0, v''_i(\cdot) < 0 \) for \( i = L, H \).

The firm does not know whether the buyer is one high or low valuation. The a priori probability that they are of a high valuation is \( \lambda \).

The firm offers the following deal

\[ \{(q_L, p_L); (q_H, p_H)\} \]
The incentive compatibility constraints are

\begin{align*}
\text{(IC}^H) & \quad v_H(q_H) - p_H q_H & \geq & \quad v_H(q_L) - p_L q_L \\
\text{(IC}^L) & \quad v_L(q_L) - p_L q_L & \geq & \quad v_L(q_H) - p_H q_H
\end{align*}
The individual rationality constraints are

$\text{(IR}^H) \quad v_H(q_H) - p_H q_H \geq 0$

$\text{(IR}^L) \quad v_L(q_L) - p_L q_L \geq 0$
The optimal schedule solves

\[ \max \lambda \cdot p_{HqH} + (1 - \lambda) \cdot p_{LqL} \]

s.t

\[ IC^H, IC^L, IR^H \text{ and } IR^L \]
With $\lambda = 0.35$, $c = 10$, $v_H(q) = 2v_L(q)$ and

$$v_L(q) = 100\sqrt{q}$$

The optimal solution is

$$p^*_H = 64 \quad q^*_H = 6$$

$$p^*_L = 95 \quad q^*_L = 1.1$$