INCENTIVE CONTRACTS

Agency Relationship: one person’s welfare depends on what another person does.

Agent is the person who acts. Principal is the person whom the action effects. For instance:

- Company owners are principals.
- Workers and managers are agents.
- Owners do not have complete knowledge.
- Employees may pursue their own goals and reduce profits.
PRINCIPAL-AGENT PROBLEM (PRIVATE)
The Principal–Agent Problem in Private Enterprises:

- Only 16 of 100 largest corporations have individual family or financial institution ownership exceeding 10%.
- Most large firms are controlled by management.
- Monitoring management is costly (asymmetric information).
- Managers may pursue their own objectives (growth, utility from job)
• Limitations to managers’ ability to deviate from objective of owners
  - Stockholders can oust managers
  - Takeover attempts
  - Market for managers who maximize profits
PRINCIPAL-AGENT PROBLEM (PUBLIC)

- Managers’ goals may deviate from the agencies goal (size)
- Oversight is difficult (asymmetric information)
- Market forces are lacking
INCENTIVE CONTRACTS
Designing a reward system to align the principal and agent’s goals—an example
- Watch manufacturer
- Uses labor and machinery
- Owners goal is to maximize profit
Machine repairperson can influence reliability of machines and profits
EXAMPLE
Profits also depend, in part, on the quality of parts and the reliability of labor. High monitoring cost makes it difficult to assess the repair-person’s work.
Repairperson can work with either high or low effort. Profits depend on effort relative to the other events (poor or good luck, equally likely)

<table>
<thead>
<tr>
<th></th>
<th>Poor Luck</th>
<th>Good Luck</th>
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</thead>
<tbody>
<tr>
<td>Low effort ((e = 0))</td>
<td>$10</td>
<td>$20</td>
</tr>
<tr>
<td>High effort ((e = 1))</td>
<td>$20</td>
<td>$40</td>
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Owners cannot determine a high or low effort when profits = $20. Repairperson’s goal is to maximize wage net of cost. His/her cost structure is:

<table>
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<th>Effort</th>
<th>Cost</th>
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Outside opportunities ensure a reservation profit of $2
EXAMPLE (CHOOSING A WAGE)

\( w(R) = \) repairperson wage based only on observed profits

\[
egin{align*}
R &= $10 \quad w(R) = 0 \\
R &= $20 \quad w(R) = 0 \\
R &= $40 \quad w(R) = $24
\end{align*}
\]

The expected wage under effort

\[
E[w(R)| e = 1] - C(1) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 24 - 10 = 2
\]
CONCLUSION
The agent has an incentive to engage in effort, for

\[ E[w(R)| e = 1] - C(1) > E[w(R)| e = 0] - C(0) \]

From the principal’s perspective:

\[ E[R - w(R)| e = 1] = \frac{1}{2}20 + \frac{1}{2}(40 - 24) = 18 \]

\[ > \]

\[ E[R - w(R)| e = 0] = \frac{1}{2}10 + \frac{1}{2}20 = 15 \]
Incentive structure that rewards the outcome of high levels of effort can induce agents to aim for the goals set by the principals.
PRINCIPAL-AGENT GAMES UNDER PERFECT INFORMATION

Assuming perfect information provides a benchmark for the more realistic cases (information asymmetry). The principal moves first and can either offer a contract or not. If the principal offers a contract, it must specify what wages will be paid if a *high* effort is observed and if a *low* effort is observed.
The contract is a schedule of the form

\[ \{(e_H, R(e_H)); (e_L; R(e_L))\} \]

The Principal’s utility is

\[ R(e) - w(e) \]

for \( e \in \{e_L, e_H\} \) and \( e_H > e_L \).
Offer contract

Accept

High Effort

(R(e_H) - w(e_H), w(e_H)-e_H)

Reject

Low Effort

(R(e_L) - w(e_L), w(e_L)-e_L)

Don’t

(R(0),0)

(R(0),0)
The agent’s utility is

\[ w(e) - e \]

where we assume a linear effort cost. For a given contract, we solve the game via backward induction:

- For the agent to rationally engage in a high effort level, it must be true that

\[ w(e_H) - e_H > w(e_L) - e_L \]
This is the *incentive compatibility* constraint.

- For the agent to accept the contract we must have

\[ w(e_H) - e_H > \text{Reservation Utility} \]

This is the *participation constraint.*
• Finally, for the principal to offer the contract is must be true that

\[ R(e_H) - w(e_H) \]

• Summarizing, the contract design must satisfy (in order to induce high effort)

\[ w(e_H) - e_H > w(e_L) - e_L \]

\[ w(e_H) - e_H > 0 \]
IMPERFECT INFORMATION

Let us assume now that whenever an agent engages in high effort level, revenue is a random variable of the form

\[ R(e_H) = \begin{cases} 20 & \text{with prob. 0.9} \\ 10 & 0.1 \end{cases} \]

Similarly,

\[ R(e_L) = \begin{cases} 20 & \text{with prob. 0.1} \\ 10 & 0.9 \end{cases} \]

where \( e_H = 8 \) and \( e_L = 2 \).
CONTRACT DESIGN
The agent’s expected value for high effort is
\[0.9 \cdot w(20) + 0.1 \cdot w(10) - e_H\]
Similarly, for low effort
\[0.1 \cdot w(20) + 0.9 \cdot w(10) - e_L\]
Incentive compatibility requires
\[0.9 \cdot w(20) + 0.1 \cdot w(10) - e_H \geq 0.1 \cdot w(20) + 0.9 \cdot w(10) - e_L\]

(IC)
High Effort

Accept

Offer contract

Low Effort

Reject

Don’t

(R(0),0)

(20 - w(20), w(20)-e_H)

0.9

(20 - w(20), w(20)-e_L)

0.1

(10 - w(10), w(10)-e_H)

0.1

(10 - w(10), w(10)-e_L)

0.9

(R(0),0)

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This is equivalent to

\[ w(20) \geq w(10) + 7.5 \]

The participation constraint requires

\[ 0.9 \cdot w(20) + 0.1 \cdot w(10) - 8 \geq 0 \]
Let us consider the contract

\[ w(10) = 0 \]
\[ w(20) = 9 \]

The principal wants to maximize

\[ 0.9 \cdot (20 - w(20)) + 0.1 \cdot (10 - w(10)) \]

subject to IC and IR constraints.
\[ w(20) = w(10) + 7.5 \]
\[ w(20) = 8.9 - 0.11w(10) \]
In fact, we can check that $w(10) = 0$ and $w(20) = 9$ is the optimal contract with an expected payoff of

$$0.9 \cdot (11) + 0.1 \cdot (10) = 10.9$$
REGULATORY GAMES
There are two players in this game, player 1 is the utility company and player 2 is the regulatory agency. Demand for service is a random variable:

\[
D = \begin{cases} 
1 & \text{with probability } 1 - q \\
\frac{1}{2} & \text{with probability } q 
\end{cases}
\]

Before uncertainty is resolved, the utility must invest to setup a capacity of \( K \). The output is therefore

\[
D(K) = \min\{K, D\}
\]
Assuming a constant marginal production cost of $c$, the utility’s profit is

$$U(R, D) = R - cD(K) - rK$$

where $r$ is the cost of capital and $R$ is the regulated revenue. We shall assume the regulatory agency aims at maximizing consumer surplus. Given a constant marginal “willingness to pay” $v > c$, the regulatory agency’s objective is to maximize

$$W(R, D) = vD(K) - R$$
Depending upon \( R \), the optimal capacity decision by the utility is 0, \( \frac{1}{2} \) and 1.

Installing capacity to cover peak demand is socially efficient when

\[
(1 - q)(v - c) > r
\]

If the regulatory agency allows a charge of \( p \in [c, v] \) per unit, the resulting decision problem for the utility is illustrated in Figure 1.
In order to induce an investment level of $K = 1$, the allowed charge must be such that

$$(1 - q)(p - c) + q \frac{1}{2}(p - c) - r \geq \frac{1}{2}(p - c - r)$$

Or equivalently,

$$p \geq c + \frac{r}{1 - q}$$
Fig 1: Regulation Game

\[
(0.5(p - c - r); 0.5(v - p ))
\]

\[
(0.5(p - c - r); 0.5(v - p ))
\]

\[
(0.5(p - c - r); 0.5(v - p ))
\]

\[
(0 ; 0 )
\]
INCENTIVE REGULATION

- Let us suppose now the firm is informed of the realization of demand but the regulator is not.
- Cost of capital $r(K)$ is increasing in investment level $K$, i.e.

$$r(K = 1) \geq r(K = \frac{1}{2})$$
To model this situation, we need to modify the regulatory game (see Fig 2):

- The regulator sets the rate before demand is revealed to the firm
- The firm decides on the investment level
Fig 2: Regulatory Game with Asymmetric Information

\[ (p - c - r(1); v - p) \]
\[ (0.5(p - c) - r(1)); 0.5(v - p)) \]
\[ (0.5(p - c - r(0.5)); 0.5(v - p)) \]
\[ (0.5(p - c - r(0.5)); 0.5(v - p)) \]
With a regulated rate $p$, when demand is high, i.e. $D = 1$ there is an incentive to *underinvest*, i.e $K = 1$ iff:

$$p - c - r(1) < 0.5(p - c - r(0.5))$$

or equivalently,

$$p - c < 2r(1) - r(0.5)$$
The regulator can avoid this socially inefficient outcome by setting a menu of regulated rates contingent on investment levels undertaken, say \( p(K) \).

The incentive compatibility constraint becomes

\[
p(1) - c - r(1) \geq 0.5(p(0.5) - c - r(0.5))
\]
The optimal menu is therefore

\[ p(0.5) = c + r(0.5) \]

\[ p(1) = c + r(1) + \text{incentive} \]