REGULATION UNDER UNCERTAINTY
We assume the current (per unit) cost of capital is $r_0$ and
• the quantity demanded $D$ is uncertain and insensitive to the regulated price $p$.
• installed capacity is $K$ and per unit operating cost is $c$.
• the (uncertain) per unit cost of capital at the end of one period is $r_1$, and physical depreciation is denoted by $d$.
In light of the above setup, the random value of the firm after one period is:

\[ X = (p - c) \min \{D, K\} + r_1(1 - d)K \]  

The question now posed is, what is today’s market value?
CAPITAL ASSET PRICING MODEL (CAPM)

The expected return of any security $i$ satisfies

$$E[r_i] = r + \frac{Cov(r_i, r_m)}{Var(r_m)}(E[r_m] - r)$$

where $r$ is the risk free interest rate and $r_m$ is the (random) return on the market portfolio. Sometimes this equation is written in terms of yield (say $X_i$), where

$$r_i = \frac{X_i - V}{V}$$
where $V$ denotes the security’s market value. It follows that

$$V = \frac{E(X_i) - \lambda Cov(X_i, r_m)}{1 + r}$$

where

$$\lambda = \frac{E[r_m] - r}{Var(r_m)}$$

is referred to as the market price of risk.
BACK TO REGULATION

We shall apply the CAPM valuation to equation (1) we obtain the market value of the regulated firm. For simplicity let us first analyze the case where \( d = 1 \) and let us denote by \( D(K) = \min\{D, K\} \):

\[
V^1 = \frac{E[X] - \lambda \text{Cov}(X, r_m)}{1 + r} = \frac{p - c}{1 + r} [E[D(K)] - \lambda \text{Cov}(D(K), r_m)]
\]
For a given choice of $K$, and assuming the firm has no debt the regulated price $p^1(K)$ must satisfy:

$$r_0K = \frac{p^1(K) - c}{1 + r} \left[ E[D(K)] - \lambda \text{Cov}(D(K), r_m) \right]$$

From which it follows that

$$p^1(K) = c + \frac{(1 + r)r_0K}{E[D(K)] - \lambda \text{Cov}(D(K), r_m)}$$
CONCLUSIONS

- If there is no market risk, i.e. $Cov(D, r_m) = 0$ and $K > D$, equation (2) simply asserts that

$$p^1(K) = c + (1 + r)r_0 \frac{K}{E[D]}$$

where $\frac{K}{E[D]}$ is a “premium” for excess capacity.
• If there is market risk, i.e. $Cov(D, r_m) > 0$, and $K > D$, equation (2) implies

$$p^1(K) = c + (1 + r)r_0\frac{K}{E[D] - \lambda Cov(D, r_m)}$$

In words, the premium for “excess” capacity increases non-linearly with market risk.
If there is market risk, i.e. $Cov(D, r_m) > 0$ and the implemented regulated price is only \( c + (1 + r)r_0 \), then the firm must adjust its investment plan such that

\[
K = E[D]
\]

Note here that this is not a choice of the firm but a reaction induced by the capital market.
THE EFFECT OF RATE BASE VALUE

Let us now assume $d = 0$. Amongst the many rate base accounting methods, when applied to our simple example we would have that the rate base value at the end of one period, say $B_1$, is:

- **Historical Cost:**
  \[ B_1 = r_0K \]

- **Replacement Cost:**
  \[ B_1 = r_1K \]
• Optimized Replacement Cost:

\[ B_1 = r_1 \min\{D, K\} = r_1 D(K) \]

• Optimized Deprival Value

\[ B_1 = \begin{cases} 
    r_1 K & K \leq D \\
    r_1 D + \text{Value}(K - D) & K > D 
\end{cases} \]
In what follows we shall continue to assume \( d = 0 \). When Replacement Cost is the rate base valuation method used, the CAPM valuation when applied to (1) yields:

\[
V^0 = V^1 + \frac{E[r_1] - \lambda \operatorname{Cov}(r_1, r_m)}{1 + r} K
\]

For a given choice of \( K \), and assuming the firm has no debt, the
regulated price \( p^0(K) \) must satisfy:

\[
r_0 K = V^0
\]

\[
= \frac{p^0(K) - c}{1 + r} \left[ E[D(K)] - \lambda \text{Cov}(D(K), r_m) \right] + \frac{E[r_1] - \lambda \text{Cov}(r_1, r_m)}{1 + r} K
\]
From which it follows that

\[ p^0(K) = c + \frac{(1 + r)r_0 - E[r_1] + \lambda Cov(r_1, r_m)}{E[D(K)] - \lambda Cov(D(K), r_m)}K \]
CONCLUSIONS
Let us assume there is no market risk, i.e. $\text{Cov}(D, r_m) = 0$ (it follows that $\text{Cov}(r_1, r_m) = 0$ as well) and $K > D$, equation (2) simply asserts that

$$p^0(K) = c + ((1 + r)r_0 - E[r_1]) \frac{K}{E[D]} < p^1(K)$$

where $\frac{K}{E[D]}$ is a “premium” for excess capacity.
Consider now the case where \( r_1 = r_0 \) and Optimized Replacement Cost is the rate base valuation method used. The CAPM valuation when applied to (1) yields:

\[
V^0 = V^1 + \frac{r_0}{1 + r}[E[D(K)] - \lambda \text{Cov}(D(K), r_m)]
\]

For a given choice of \( K \), and assuming the firm has no debt, the regulated price \( p^0(K) \) must satisfy:

\[
r_0K = \frac{p^0(K) - c}{1 + r}[E[D(K)] - \lambda \text{Cov}(D(K), r_m)]
\]
From which it follows that

\[ p^0(K) = c - r_0 + \frac{(1 + r) r_0}{E[D(K)] - \lambda \text{Cov}(D(K), r_m)} K \]  

(4)

Here again, we note the effect of demand risk borne by the firm in the form of a “premium” over excess capacity.