MONOPOLY

• An industry is a *natural* monopoly if the provision of a particular good or service by a single firm minimizes cost.
• Alternatively, due to *legal* constraints, a firm may also enjoy a monopoly status.
• Technological improvements may drastically change the cost structure of a given industry.
A SIMPLE TEST
Suppose the cost structure is composed of a fixed cost, say $F$ and a marginal cost $C''(q)$. Then total production cost is

$$C(q) = F + \int_{0}^{q} C''(q)$$

For example, if $C''(q) = \alpha + 2\beta q$ we have

$$C(q) = F + \alpha q + \beta q^2$$
The average cost is

$$AC(q) = \frac{C(q)}{q} = \frac{F}{q} + \alpha + \beta q$$

If there are two symmetric firms (each producing $\frac{q}{2}$), then the total cost is:

$$C_2(q) = F + \alpha \frac{q}{2} + \beta \left( \frac{q}{2} \right)^2 + F + \alpha \frac{q}{2} + \beta \left( \frac{q}{2} \right)^2$$
Thus,

\[ AC_2(q) = \frac{C_2(q)}{q} = \frac{2F}{q} + \alpha + \beta \frac{q}{2} \]

Let \( q^* \) denote the output level at which \( AC(q^*) = AC_2(q^*) \)

\[ q^* = \sqrt{\frac{2F}{\beta}} \]
A simple test
Note that

$$AC_2(q) > AC'(q) \text{ for } q \leq q^*$$

Hence, if the forecasted demand is less than $q^*$, society will be better served by having a single firm provide the good or service.
ECONOMIES OF SCALE

There are economies of scale, whenever

\[
\frac{\text{average cost at } q}{\text{marginal cost at } q} > 1
\]

For example, when \( C(q) = F + cq \) we have

\[
\rho(q) = \frac{F}{q} + c = 1 + \frac{F}{cq}
\]

Thus, the greater the fixed cost the greater the economies of scale.
Average cost is greater than marginal cost if and only if average cost is decreasing.
To see this note that for $dq > 0$

\[
\frac{C(q + dq)}{q + dq} \leq \frac{C(q)}{q}
\]

which implies

\[
\frac{C(q + dq) - C(q)}{dq} \leq \frac{C(q)}{q}
\]
As $dq \to 0$ we have

$$\frac{dC(q)}{dq} \leq AC(q)$$
SOCIALLY OPTIMAL PRICING

- We aim at maximizing social surplus; i.e. the sum of consumer and producer surplus.
- For a given demand function $D(p)$, we refer to $p(Q)$ as the inverse demand function. For example, if $D(p) = \bar{D} - bp$ then

\[
p(Q) = \frac{1}{b}[\bar{D} - Q]
\]
Suppose price is set at $p^*$. Then, the consumers with willingness to pay $p > p^*$ experience a (per unit) surplus $p - p^*$. Thus, the consumer surplus is

$$CS(p^*) = \int_0^{D(p^*)} [p(Q) - p^*]dQ$$

Note that

$$\frac{dCS(p^*)}{dp^*} = -D(p^*) < 0$$

Hence, consumer surplus is monotone decreasing in price.
Consumer Surplus

\[ P(Q) \]

\[ p = p^* \]
Producer surplus is

\[ PS(p^*) = p^* D(p^*) - C(D(p^*)) \]

Thus,

\[ \frac{dPS(p^*)}{dp^*} = D(p^*) + p^* \frac{dD(p^*)}{dp} - C'() \frac{dD(p^*)}{dp} \] (2)
The socially optimal price is the solution to

\[
\max_p [CS(p) + PS(p)]
\]

The first order condition, using (1) and (2) is

\[
-D(p^*) + D(p^*) + p^* \frac{dD(p^*)}{dp} - C'(\cdot) \frac{dD(p^*)}{dp} = 0
\]
This implies

\[(p^* - C''(\cdot)) \frac{dD(p^*)}{dp} = 0\]

Or equivalently,

\[p^* = C''(\cdot)\]

In words, *socially optimal price equals marginal production cost.*
MONOPOLY PRICING
If left to its own devices the monopolist will appropriate society’s gain from the monopoly status.
The monopolist price $p^m$ solves the problem
$$\max_p[pD(p) - C(D(p))]$$
The first order condition is
$$D(p^m) + p^m \frac{dD(p^m)}{dp} = C'(\cdot) \frac{dD(p^m)}{dp}$$
Or equivalently,
$$p^m - C'(\cdot) = \frac{D(p^m)}{\frac{dD(p^m)}{dp}}$$
Dividing on both sides by $p^m$

$$
\frac{p^m - C'(\cdot)}{p^m} = \frac{1}{\epsilon}
$$

where $\epsilon$ is the price-elasticity of demand

$$
\epsilon = -\frac{dD(p)}{D(p)} \frac{dp}{p}
$$

In conclusion, the monopolist optimal price-cost margin is decreasing in the price elasticity of demand.
• Since social surplus (consumer + producer surplus) is maximized at marginal cost, monopoly pricing induces welfare losses.

• The higher the markup over marginal cost the higher the welfare loss.

• This is why natural monopolies are to be regulated.
REGULATION
As we have shown above, if left unchecked, the monopolist will appropriate society’s gain from the monopoly status. The term “economic regulation” refers to the rules governing the performance of a monopolist. One of the simplest approaches is known as “marginal cost” regulation.
Suppose the firms cost structure is

\[ C(q) = F + cq \]

and let us assume that the regulated charge for the good or service \( p^R \) is set to equal \( c \).

Note that the firm is to incur is losses for

\[ cD(c) - F - cD(c) = -F \]

To guarantee solvency the government makes a transfer to the firm in the amount \( F \).
AVERAGE COST REGULATION
Transfer from the government are costly. The economy has to be taxed and distortions are therefore induced. Moreover, the overall effectiveness of the scheme may be in doubt for governments tend to behave in a fiscally irresponsible manner (this is just a by-product of the democratic system).
This mechanism also induces “rent seeking”. The regulated firm engages in wasteful lobbying in order to increase the transfer value.
Regulatory Agency

Firm

Consumers

Government

Tax Agency

F

$p^R = c$

c \times D(c)

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An alternative to marginal cost is average cost regulation.

\[ p^A = AC'(q^A) \]
\[ q^A = D(p^A) \]

In theory, this is a “second best” mechanism for \( c < p^A \) (or equivalently, \( D(p^A) < D(c) \))