HW#6 (Due Oct 9). Points [5 + 5 + 10]

In this problem we are going to write several lines of code to numerically solve a diffusion equation. Consider a bar of semiconductor material with length L. In the beginning we shine a beam of light onto the middle of the material (x=L/2) and the generated photons are all absorbed which further give rise to a certain amount of extra carriers. After that we turn the light off so there will be no light thereafter. We would like to know the behavior of generated minority carriers.

In order to simplify the problem, we assume that the recombination time of minority carriers is negligible. Also we assume we are dealing with an 1-D material in which the transverse diffusion is ignored. So according to MCDE, the minority carrier distribution is governed by equation:

\[
\frac{\partial \Delta n}{\partial t} = D \frac{\partial^2 \Delta n}{\partial x^2}
\]

in which \( \Delta n \) is the density of extra minority carriers, \( t \) is the time, \( x \) is the 1-D spatial coordinate, \( D \) is the diffusion constant. Now we will numerically solve this equation.

(a) Note that in the equation \( x \), \( t \) and \( D \) all have units. When dealing with these PDE numerically, it is always useful to scale all these parameters to get rid of physical units, because then we can concentrate on the dimensionless equation itself without messing around all the units. Show that given \( L \) and \( D \), the equation can be scaled into a dimensionless one in form of:

\[
\frac{\partial \Delta n}{\partial \gamma} = \frac{\partial^2 \Delta n}{\partial \rho^2}
\]

and state the meaning of \( \gamma \) and \( \rho \) here.

(b) Now we will solve the dimensionless diffusion equation explicitly. We first uniformly sample the spatial coordinate with \( N \) points (with step \( \delta \rho \)), and write diffusion part in the right side in form of a \( N \) by \( N \) matrix multiplied by a column vector which stands for the carrier distribution in 1-D grid. For the left side, the time step is \( \delta \gamma \). Show that in order to have a stable solution (which means the solution is converged in the end and total carrier number will stay physically meaningful), \( \delta \gamma \) and \( \delta \rho \) must satisfy:

\[
\delta \gamma < 0.5 \delta \rho^2.
\]

(c) Usually it is very important to set up boundary condition properly for a PDE. But in this case, since the initial generated carriers are far from boundaries it is not so important within certain time. Use either box boundary condition or periodic boundary condition with an initial condition defined in this problem to solve this equation, and show the change of carrier spatial distribution with time.