What We Don’t Know Doesn’t Hurt Us: Rational Inattention and the Permanent Income Hypothesis in General Equilibrium

Yulei Luo†  Gaowang Wang‡
The University of Hong Kong  Central University of Finance and Economics
Eric R. Young§
The University of Virginia
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Abstract

This paper derives the general equilibrium effects of rational inattention (Sims 2003) in a model of incomplete income insurance (Huggett 1993, Wang 2003). We show that, under the assumption of CARA utility with Gaussian shocks, the Permanent Income Hypothesis (PIH) arises in equilibrium, as in models with full information-rational expectations, due to a balancing of precautionary savings and impatience. We then show that the welfare costs of incomplete information are even smaller due to general equilibrium adjustments in interest rates.

Keywords: Rational Inattention; Permanent Income Hypothesis; Aggregate Saving, General Equilibrium

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†School of Economics and Finance, Faculty of Business and Economics, The University of Hong Kong, Hong Kong. E-mail: yluo@econ.hku.hk.
‡China Economics and Management Academy, Central University of Finance and Economics, Beijing, China. E-mail: wanggaowang@gmail.com.
§Department of Economics, University of Virginia, Charlottesville, VA 22904. E-mail: ey2d@virginia.edu.
1. Introduction

In intertemporal consumption-savings problems, households save today for three reasons: (i) they anticipate future declines in income, (ii) they face uninsurable risk that generates precautionary savings, and (iii) they are relatively patient compared to the interest rate. When only motive (i) is operative then one obtains the “permanent income hypothesis (PIH)” of Friedman (1957), where consumption is solely determined by permanent income and follows a random walk. The PIH has some implications that are strongly inconsistent with the data. Two implications in particular are discussed in Campbell and Deaton (1989), the excess sensitivity and excess smoothness puzzles. Excess sensitivity occurs if consumption responds to predictable changes in income; under the PIH those changes are part of permanent income and therefore have already had their effect on consumption. Excess smoothness occurs if consumption responds less than one for one to permanent changes in income (or equivalently less than one for one to changes in permanent income). The two puzzles are actually manifestations of the same underlying economic forces, as shown in Campbell and Deaton (1989), and their absence is profoundly rejected. Unfortunately, uninsurable income risk seems to be pervasive in microeconomic data, and general equilibrium models with uninsurable risk tend to predict impatience of households (that is, they face “low interest rates”), so that the basic PIH is violated.

Wang (2003), using a simple model with CARA utility and risk free assets in zero net supply, shows that the PIH reemerges in general equilibrium – when decision rules are linear, the equilibrium interest rate exactly balances the forces of precautionary saving and dissaving due to impatience, even in the presence of uninsurable risk. Due to the linearity of consumption as a function of individual permanent income, Wang (2003) is able to analytically characterize the forces that operate in general equilibrium and show they cancel out, under some mild assumptions about the labor income process.

Luo (2008) and Luo and Young (2010) introduce rational inattention into the basic partial equilibrium PIH environment; RI implies that agents process signals slowly and therefore appear to respond sluggishly to innovations in permanent income. This sluggish response appears to deliver changes in consumption in response to anticipated income changes, because econometricians actually observe more than the agents do, and as a result also delivers smaller responses to permanent income changes.

Our goal in this paper is to ask the same question from Wang (2003) – namely, does the PIH
reemerge in general equilibrium – in the presence of rational inattention. We study economies with constant absolute risk aversion (CARA) preferences, as they simultaneously generate precautionary savings and linear consumption rules, and characterize the forces that act on the general equilibrium interest rate. We find that the PIH does describe equilibrium consumption behavior in general equilibrium, with the appropriate substitution of actual permanent income by perceived permanent income. Thus, the delicate canceling of precautionary and impatience forces found by Wang (2003) carries over unmodified to models with incomplete information about the state.\footnote{Luo and Young (2013) document a observational equivalence between rational inattention and signal extraction in linear-quadratic-Gaussian models.}

One key result in this paper is that there exists general equilibrium interest rates clearing the asset market and they are significantly affected by the degree of RI. After obtaining the explicit expression for consumption dynamics, we examine how RI affects the stochastic properties of the joint dynamics of consumption growth to income growth. Specifically, we find that the effect of RI on consumption dynamics is attenuated by general equilibrium adjustment in the interest rate – as processing capacity declines the interest rate also declines, leading to lower consumption volatility. The implication is that the costs of incomplete information have likely been overestimated in the literature, despite being very tiny to start.\footnote{For the welfare losses due to imperfect information about current income or permanent income calculated in the partial equilibrium linear-quadratic (LQ) permanent income models, see Pischke (1995), Luo and Young (2010), and Luo, Nie, and Young (2014).}

This paper is organized as follows. Section 2 constructs a precautionary saving model with a continuum of inattentive consumers who have the CARA utility and face uninsurable labor income. Section 3 solves optimal consumption-saving rules under rational inattention and examines the general equilibrium effects of RI on the interest rate and the joint dynamics of consumption and income. Section 4 discusses the case with fixed information-processing cost and elastic capacity. Section 5 concludes.


2.1. A Full-information Rational Expectations Model with Precautionary Savings

Following Caballero (1990) and Wang (2003), we formulate a full-information rational expectations (FI-RE) model with precautionary savings as follows:

\[
\max_{\{c_t\}} \mathcal{U}(c) = E_0 \left[ \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t u(c_t) \right], \tag{1}
\]
subject to the flow budget constraint

\[ a_{t+1} = (1 + r) a_t + y_t - c_t, \]  

(2)

where \( u(c_t) = -\exp(-\alpha c) / \alpha \) is a constant-absolute-risk-aversion utility with \( \alpha > 0 \), \( \rho > 0 \) is the agent’s subjective discount rate, \( r \) is a constant rate of interest, and labor income, \( y_t \), follows a stationary AR(1) process with Gaussian innovations:

\[ y_t = \phi_0 + \phi_1 y_{t-1} + w_t, \quad t \geq 1, \quad |\phi_1| < 1 \]

(3)

where \( w_t \sim N(0, \sigma^2) \), \( \phi_0 = (1 - \phi_1) \bar{y}, \) \( \bar{y} \) is the mean of \( y_t \), and the initial levels of labor income \( y_0 \) and asset \( a_0 \) are given.\(^3\) Solving (1) subject to (2) and (3) yields the following optimal consumption plan:

\[ c_t = r \left\{ a_t + h_t + \frac{1}{\alpha r^2} \left[ \ln \left( \frac{1 + \rho}{1 + r} \right) - \ln E_t \left[ \exp (-r \alpha \phi w_{t+1}) \right] \right] \right\}, \]

(4)

where

\[ h_t \equiv \frac{1}{1 + r} E_t \left[ \sum_{j=0}^{\infty} \left( \frac{1}{1 + r} \right)^j y_{t+j} \right], \]

(5)

is human wealth defined as the discounted expected present value of current and future labor income and is equal \( h_t = \phi (y_t + \phi_0 / r) \) after substituting (3) into (5), and \( \phi = 1 / (1 + r - \phi_1) \).\(^4\)

This consumption function is the same as that obtained in Wang (2003). In the last two terms in (4), \( \ln \left( \frac{1 + \rho}{1 + r} \right) / \alpha r \) measures the relative importance of impatience and the interest rate in determining current consumption, and \( \ln E_t \left[ \exp (-r \alpha \phi w_{t+1}) \right] / \alpha r \) measures the amount of precautionary savings determined by the interaction of risk aversion and income uncertainty.

In order to facilitate the introduction of rational inattention we follow Luo (2008) and Luo and Young (2010) and reduce the multivariate model to a univariate model with iid innovations to permanent income. Letting permanent income, \( s_t = a_t + h_t \), be defined as a new state variable, we can reformulate the PIH model as

\[ v(s_t) = \max_{c_t} \left\{ u(c_t) + \frac{1}{1 + \rho} E_t [v(s_{t+1})] \right\}, \]

\(^3\)It is worth noting that assuming that the individual income shock includes two components, one is permanent and the other is transitory, does not change the main results in this paper. Here we follow Wang (2003) to adopt specification (3). The detailed derivation of the model with the two-income shock specification is available from the corresponding author by request. For the empirical studies on the income specification, see Attanasio and Pavoni (2007).

\(^4\)See Appendix 6.1 for the derivation.
subject to

\[ s_{t+1} = (1 + r) s_t - c_t + \xi_{t+1}, \]  

(6)

where the time \((t + 1)\) innovation to permanent income can be written as

\[ \xi_{t+1} = \frac{1}{1 + r} \sum_{j=t+1}^{\infty} \left( \frac{1}{1 + r} \right)^{j-(t+1)} (E_{t+j} - E_t) [y_j], \]  

(7)

which can be reduced to \( \xi_{t+1} = \phi w_{t+1} \) when we use the income specification, (3), where \( v(s_t) \) is the consumer’s value function under FI-RE.\(^5\)

2.2. Incorporating Rational Inattention

In this section, we follow Sims (2003) and incorporate rational inattention (RI) due to finite information-processing capacity into the above permanent income model with the CARA-Gaussian specification. Under RI, consumers have only finite Shannon channel capacity to observe the state of the world. Specifically, we use the concept of entropy from information theory to characterize the uncertainty about a random variable; the reduction in entropy is thus a natural measure of information flow.\(^6\) With finite capacity \( \kappa \in (0, \infty) \), a random variable \( \{s_t\} \) following a continuous distribution cannot be observed without error and thus the information set at time \( t + 1 \), denoted \( I_{t+1} \), is generated by the entire history of noisy signals \( \{s^*_j\}_{j=0}^{t+1} \). Following the literature, we assume the noisy signal takes the additive form

\[ s^*_{t+1} = s_{t+1} + \xi_{t+1}, \]

where \( \xi_{t+1} \) is the endogenous noise caused by finite capacity.\(^7\) We further assume that \( \xi_{t+1} \) is an iid idiosyncratic shock and is independent of the fundamental shocks hitting the economy. The reason that the RI-induced noise is idiosyncratic is that the endogenous noise arises from the consumer’s own internal information-processing constraint. Agents with finite capacity will choose a new signal \( s^*_{t+1} \in I_{t+1} = \{s^*_1, s^*_2, \ldots, s^*_{t+1}\} \) that reduces the uncertainty about the variable \( s_{t+1} \) as much

\(^5\)See Appendix 6.1 for the derivation.

\(^6\)Formally, entropy is defined as the expectation of the negative of the (natural) log of the density function, \(-E[\ln(f(s))]\). For example, the entropy of a discrete distribution with equal weight on two points is simply \( E[\ln(f(s))] = -0.5 \ln(0.5) = 0.69 \), and the unit of information contained in this distribution is 0.69 “nats”. (For alternative bases for the logarithm, the unit of information differs; with log base 2 the unit of information is the ‘bit’ and with base 10 it is a ‘dit’ or a ‘hartley.’) In this case, an agent can remove all uncertainty about \( s \) if the capacity devoted to monitoring \( s \) is \( \kappa = 0.69 \) nats.

as possible. Formally, this idea can be described by the information constraint

\[ H(s_{t+1}|I_t) - H(s_{t+1}|I_{t+1}) \leq \kappa, \tag{8} \]

where \( \kappa \) is the investor’s information channel capacity, \( H(s_{t+1}|I_t) \) denotes the entropy of the state prior to observing the new signal at \( t + 1 \), and \( H(s_{t+1}|I_{t+1}) \) is the entropy after observing the new signal. \( \kappa \) imposes an upper bound on the amount of information flow – that is, the change in the entropy – that can be transmitted in any given period. Finally, following the literature, we suppose that the prior distribution of \( s_{t+1} \) is Gaussian.

Although we adopt the CARA-Gaussian setting in our model, we will assume the loss function due to imperfect-state-observation is still quadratic. Using a quadratic loss function, Sims (2003) shows that the true state under RI also follows a normal distribution \( s_t|I_t \sim N(E[s_t|I_t], \Sigma_t) \), where \( \Sigma_t = E_t \left[ (s_t - \hat{s}_t)^2 \right] \). In addition, given that the noisy signal takes the additive form \( s^*_t = s_{t+1} + \xi_{t+1} \), the noise \( \xi_{t+1} \) will also be Gaussian. In this case, (8) reduces to

\[ \ln(|\Psi_t|) - \ln(|\Sigma_{t+1}|) \leq 2\kappa, \tag{9} \]

where \( \Sigma_{t+1} = \text{var}_{t+1}(s_{t+1}) \) and \( \Psi_t = \text{var}_t(s_{t+1}) = (1 + r)^2 \Sigma_t + \text{var}_t(\xi_{t+1}) \) are the posterior and prior variance of the state variable, \( s_{t+1} \), respectively. In our univariate model, (9) fully determines the value of the steady state conditional variance \( \Sigma \):

\[ \Sigma = \frac{\text{var}_t(\xi_{t+1})}{\exp(2\kappa) - (1 + r)^2}, \tag{10} \]

which means that \( \Sigma \) is entirely determined by the variance of the exogenous shock (\( \text{var}_t(\xi_{t+1}) \)) and finite capacity (\( \kappa \)).\(^8\) Following the steps outlined in Luo and Young (2013), we can compute the Kalman gain in the steady state \( \theta \) as

\[ \theta = 1 - 1/\exp(2\kappa); \tag{11} \]

\( \theta \) measures the fraction of uncertainty removed by a new signal in each period.\(^9\)

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\(^8\)Note that here we need to impose the restriction \( \exp(2\kappa) - (1 + r)^2 > 0 \). If this condition fails, the state is not stabilizable and the unconditional variance diverges.

\(^9\)One could instead model RI as having a fixed marginal cost of acquiring channel capacity; as discussed in Luo and Young (2013) the two formulations are equivalent for the questions at hand here. See Section 4 for a detailed discussion on this case.
The evolution of the estimated state \( \hat{s}_t \) is governed by the Kalman filtering equation

\[
\hat{s}_{t+1} = (1 - \theta) ((1 + r) \hat{s}_t - c_t) + \theta s_{t+1}^*.
\]

(12)

Combining (6) with (12) yields

\[
\hat{s}_{t+1} = (1 + r) \hat{s}_t - c_t + \hat{s}_{t+1},
\]

(13)

where

\[
\hat{\xi}_{t+1} = \theta (1 + r) (s_t - \hat{s}_t) + \theta (\xi_{t+1} + \zeta_{t+1})
\]

(14)

is the innovation to \( \hat{s}_{t+1} \) and is independent of all the other terms on the RHS of (13). \( \hat{\zeta}_{t+1} \) is an MA(\( \infty \)) process with \( E_t [\hat{\xi}_{t+1}] = 0 \) and

\[
\text{var} \left( \hat{\xi}_{t+1} \right) = \Gamma (\theta, r) \omega^2_{\xi},
\]

where \( \Gamma (\theta, r) = \frac{\theta}{1 - (1 - \theta)(1 + r)} > 1 \) for \( \theta < 1 \), and

\[
s_t - \hat{s}_t = \frac{(1 - \theta) \xi_t}{1 - (1 - \theta)(1 + r) \cdot L} - \frac{\theta \xi_t}{1 - (1 - \theta)(1 + r) \cdot L}
\]

(15)

is the estimation error with \( E_t [s_t - \hat{s}_t] = 0 \) and \( \text{var} (s_t - \hat{s}_t) = \frac{1 - \theta}{1 - (1 - \theta)(1 + r)} \omega^2_{\xi} \).

3. Main Findings

3.1. Optimal Consumption and Savings functions

Following the standard procedure in the literature, the consumption function and the value function under RI can be obtained by solving the following stochastic Bellman equation:

\[
\hat{v} (\hat{s}_t) = \max_{c_t} \left\{ -\frac{1}{\alpha} \exp (-\alpha c_t) + \frac{1}{1 + \rho} E_t [\hat{v} (\hat{s}_{t+1})] \right\},
\]

subject to (13)-(15). The following proposition summarizes the main results from the above precautionary-savings model with RI:

**Proposition 1.** For a given Kalman gain, \( \theta \), the value function is
the consumption function is
\[ c^*_t = r \left\{ \hat{s}_t + \frac{1}{\alpha r} \ln \left( \frac{1 + \rho}{1 + r} \right) - \ln E_t \left[ \exp \left( -r \hat{\zeta}_{t+1} \right) \right] \right\}, \] (18)
and the saving function is
\[ d^*_t = (1 - \phi_1) \phi (y_t - \bar{y}) + r (s_t - \hat{s}_t) + \frac{1}{r \alpha} \left[ \ln E_t \left[ \exp \left( -r \hat{\zeta}_{t+1} \right) \right] - \Psi (r) \right], \] (19)
where \( s_t - \hat{s}_t \) is an MA(∞) estimation error process given in (15) and \( \Psi (r) = \ln \left( \frac{1 + \rho}{1 + r} \right) \).

**Proof.** See Appendix 6.1.

Comparing (4) with (18), it is clear that the two consumption functions are identical except that we replace \( s_t \) with \( \hat{s}_t \) and \( \zeta_{t+1} (\equiv \phi w_{t+1}) \) with \( \hat{\zeta}_{t+1} \), respectively. First, given that
\[
\ln E_t \left[ \exp \left( -r \hat{\zeta}_{t+1} \right) \right] = \frac{1}{2} (r \alpha)^2 \omega^2_{\hat{\zeta}},
\ln E_t \left[ \exp \left( -r \hat{\zeta}_{t+1} \right) \right] = \frac{1}{2} \Gamma (\theta, r) (r \alpha)^2 \omega^2_{\hat{\zeta}},
\]
it is straightforward to show that the precautionary saving premium due to limited attention is
\[ P_{rt} = \frac{1}{\alpha r} \ln E_t \left[ \exp \left( -r \hat{\zeta}_{t+1} \right) - \exp \left( -r \hat{\zeta}_{t+1} \right) \right] = \frac{1}{2} (\Gamma (\theta, r) - 1) r \alpha \omega^2_{\hat{\zeta}}, \] (20)
which is clearly decreasing with the degree of attention (\( \theta \)), and is increasing with the coefficient of absolute risk aversion (\( \alpha \)) and the persistence and volatility of the income shock (\( \phi \) and \( \sigma^2 \)) for any given \( \theta \). In other words, RI can amplify the impact of the interaction of risk aversion and income uncertainty on increasing the amount of precautionary savings.

To further explore the precautionary savings premium in (20), we isolate the effects of RI on
individual consumption and saving by rewriting (18) as

\[ c_t^* = r \hat{s}_t + \frac{1}{ra} \left\{ \Psi (r) - \left[ \ln E_t \left[ \exp \left( -ra (1 + r) (s_t - \hat{s}_t) \right) \right] + \frac{1}{2} (ra \omega_\zeta)^2 + \frac{1}{2} (1 - \theta) \Gamma (\theta, r) (ra \omega_\zeta)^2 \right] \right\}, \tag{21} \]

where \( \Psi (r) = \ln \left( \frac{1 + \rho}{1 + r} \right) \) measures the relative importance of impatience to the interest rate in determining optimal consumption (it is greater than 0 if \( \rho > r \)),

\[ \frac{1}{ar} \ln E_t \left[ \exp \left( -ar (1 + r) (s_t - \hat{s}_t) \right) \right] = r \alpha (1 - \theta) \Gamma (\theta, r) (1 + r)^2 \omega_\zeta^2 \]

is the precautionary savings premium due to the time \( t \) estimation error, \( (ra \omega_\zeta)^2 / 2 \) is the precautionary savings premium driven by the exogenous fundamental income shocks \( \{w_t\} \), and \( (1 - \theta) \Gamma (\theta, r) (ra \omega_\zeta)^2 / 2 \) captures the precautionary savings premium driven by the endogenous noise shocks, \( \{\xi_t\} \).\(^{10}\) Note that when \( \theta \) converges to 1, the consumption function with RI, (18), reduces to that of the Wang (2003) model, (4). From (18), for finite capacity (\( \kappa < \infty \) or \( \theta \in (0, 1) \)), the precautionary saving premium due to fundamental shocks is lower than that in the full-information case, i.e., \( (ra \omega_\zeta)^2 / 2 < (ra \omega_\zeta)^2 / 2 \) because of the incomplete adjustment of consumption to the fundamental shock, while we have two new positive terms that increase the total savings more than the absolute value of the reduced savings: (i) the premium due to the estimation error and (ii) the premium due to the RI-induced endogenous noise.

Note that given the time \( t \) available information, the conditional mean of (19) can be written as:

\[ \bar{d}_t = f_t + \left( \frac{1}{2} r a \Gamma (\theta, r) \omega_\zeta^2 - \frac{1}{ra} \Psi (r) \right), \tag{22} \]

where the first term \( f_t = (1 - \phi_1) \phi (y_t - \bar{y}) \) captures the consumer’s demand for savings “for a rainy day”, and the second term, \( \frac{1}{2} r a \Gamma (\theta, r) \omega_\zeta^2 \), is the certainty equivalent of the innovation to the perceived state, \( \hat{s}_t \).

3.2. General Equilibrium under RI

As in Wang (2003), we assume that the economy is populated by a continuum of ex ante identical, but ex post heterogeneous agents, of total mass normalized to one, with each agent solving the optimal consumption and savings problem with RI specified in (16). Similar to Huggett (1993), the

\(^{10}\)This result is derived by using equation (15) and the iid property of the processes \( \{\hat{\zeta}_t\}, \{\zeta_t\}, \) and \( \{\xi_t\} \).
The risk-free asset in our model is a pure-consumption loan and is in zero net supply. The initial cross-sectional distribution of permanent income is assumed to be its stationary distribution \( \Phi (\cdot) \). By the law of large numbers in Sun (2006), provided that the spaces of agents and the probability space are constructed appropriately, aggregate permanent income and the cross-sectional distribution of permanent income \( \Phi (\cdot) \) are constant over time.

**Proposition 2.** The total savings demand “for a rainy day” in the precautionary savings model with RI equals zero for any positive interest rate. That is, \( F_t (r) = \int y_t f_t (r) d\Phi (y_t) = 0, \) for \( r > 0 \).

**Proof.** The proof uses the LLN and is the same as that in Wang (2003).

Proposition 2 states that the total savings “for a rainy day” is zero, at any positive interest rate. Therefore, from (20), for \( r > 0 \), the expression for total savings under RI in the economy at time \( t \) is

\[
D (\theta, r) \equiv \frac{1}{r^\alpha} (\Pi (\theta, r) - \Psi (r)) = \frac{1}{r^\alpha} \left[ \frac{1}{2} (r\alpha)^2 \Gamma (\theta, r) \omega_t^2 - \Psi (r) \right],
\]

where \( \Pi (\theta, r) = \frac{1}{2} (r\alpha)^2 \Gamma (\theta, r) \omega_t^2 \) measures the amount of precautionary savings, and \( \Psi (r) \) captures the dissaving effects of impatience. Given (23), an equilibrium under RI is defined by an interest rate \( r^* \) satisfying

\[
D (\theta, r^*) = 0.
\]

The following proposition shows the existence of the equilibrium and the PIH holds in the RI general equilibrium:

**Proposition 3.** There exists at least one equilibrium with an interest rate \( r^* \in (0, \rho) \) in the precautionary-savings model with RI. In any such equilibrium, each agent’s consumption is described by the PIH, in that

\[
e^*_t = r^* \hat{s}_t,
\]

where \( \hat{s}_t = E [s_t | I_t] \) is the perceived value of permanent income. Furthermore, in this equilibrium, the evolution equations of wealth and consumption are

\[
\Delta a^*_t + 1 = \frac{1 - \phi_1}{1 + r^* - \phi_1} (y_t - \overline{y}) + r^* (s_t - \hat{s}_t),
\]

\[
\Delta c^*_t + 1 = r^* \hat{\xi}_{t+1},
\]
exists at least one interest rate \( r \) individual’s optimal consumption rule under RI in general equilibrium as 

\[
c_t(\hat{t}) = \frac{\theta}{1 - (1 - \theta)(1 + r^*)}.
\]

Proof. If \( r > \rho \), the two terms, \( \Pi(\theta, r) \) and \( \Psi(r) \), in the expression for total savings \( D(\theta, r^*) \), are positive, which contradicts the equilibrium condition, \( D(\theta, r^*) = 0 \). Since \( \Pi(\theta, r) - \Psi(r) < 0 \) \((> 0)\) when \( r = 0 \) \((r = \rho)\), the continuity of the expression for total savings implies that there exists at least one interest rate \( r^* \in (0, \rho) \) such that \( D(\theta, r^*) = 0 \). From (18), we can obtain the individual’s optimal consumption rule under RI in general equilibrium as \( c_t^* = r^* \hat{s}_t \). Substituting (25) into (2) yields (26). Using (13) and (25), we can obtain (27).

The intuition behind Proposition 3 is similar to that in Wang (2003). With an individual’s constant total precautionary savings demand \( \Pi(\theta, r) \), for any \( r > 0 \), the equilibrium interest rate \( r^* \) must be at a level with the property that individual’s dissavings demand due to impatience is exactly balanced by their total precautionary-savings demand, \( \Pi(\theta, r^*) = \Psi(r^*) \). Figure 1 shows that the aggregate saving function is increasing with the interest rate, and there exists a unique interest rate \( r^* \) for every given \( \theta \) such that \( D(\theta, r^*) = 0 \).

Given (18) and (24), it is clear that even though the individual increases their total precautionary savings for information frictions, the level of aggregate savings also equals zero. That is, RI does not affect the aggregate wealth in the economy. In contrast, RI affects the equilibrium interest rate. With lower Shannon channel capacity, the equilibrium interest rate is lower. Consistent with Caballero (1991) and Wang (2003), we choose the following parameter values: \( \alpha = 1.5, \sigma = 0.15, \phi_1 = 0.95, \) and \( \rho = 0.015 \). The equilibrium interest rate is determined implicitly by the following function:

\[
\frac{1}{2} (r^*)^2 \Gamma(\theta, r^*) \omega_\sigma^2 - \ln \left( \frac{1 + \rho}{1 + r^*} \right) = 0.
\]

Using the implicit function theorem, it is straightforward to show that

\[
\frac{dr^*}{d\theta} = \frac{r^* (2 + r^*)}{\theta [1 - (1 - \theta)(1 + r^*)^2]} \left\{ \frac{2(1 - \phi_1)(1 - (1 - \theta)(1 + r^*)^2)}{r^*(1 + r^*)[1 - (1 - \theta)(1 + r^*)^2]} + \frac{1}{(1 + r^*) \ln \left( \frac{1 + \rho}{1 + r^*} \right)} \right\}^{-1} > 0,
\]

where \( 1 - (1 - \theta)(1 + r^*)^2 > 0 \) and \( \ln \left( \frac{1 + \rho}{1 + r^*} \right) > 0 \). It is clear from this expression that \( r^* \) is locally unique and decreasing in the degree of inattention \( 1 - \theta \). Figure ?? illustrates \( r^* \) as a function of \( \theta \) for different values of \( \alpha \), and clearly shows that the general equilibrium interest rate is increasing.
with the degree of attention, $\theta$.\(^{11}\)

The first row of Table 1 reports the general equilibrium interest rates for different values of $\theta$. We can see from the table that $r^*$ decreases as the degree of inattention increases. For example, if $\theta$ is reduced from 1 to 0.2, $r^*$ is reduced from 1.380 percent to 1.367 percent. One might ask what a reasonable value of $\theta$ is, and whether a drop from 1 to 0.2 is “large” in any sense. Unfortunately, it is not straightforward to answer these questions, so we simply note that 0.2 is larger than the value needed to match portfolio holdings in Luo (2010) and is therefore not obviously unreasonable.

### 3.3. Implications for Consumption and Wealth Dynamics in General Equilibrium

Luo (2008) examines how RI affects consumption volatility in a partial equilibrium version of the PIH model presented above. In general equilibrium, since RI affects the equilibrium interest rate via interacting with the coefficient of absolute risk aversion and income uncertainty, it will have an additional effect on consumption dynamics. Using (26) and (27), we can obtain the key stochastic properties of the joint dynamics of consumption and saving. The following proposition summarizes the implications of RI for the relative volatility, persistence, and correlation with income of consumption as well as the relative volatility of financial wealth to income:

**Proposition 4.** Under RI, the relative volatility of consumption to income is

$$
\mu_{cy} \equiv \frac{sd(\Delta c_{t+1})}{sd(\Delta y_{t+1})} = \phi r^* \sqrt{\frac{(1 + \phi_1) \Gamma(\theta, r^*)}{2}},
$$

(30)

the contemporaneous correlation between consumption and output is

$$
\rho_{cy} \equiv \text{corr}(\Delta c_t, \Delta y_t) = \theta \frac{1 - (1 - \theta)(1 + r^*)}{1 - (1 - \theta)(1 + r^*)} \frac{\sqrt{1 + \phi_1}}{2 \Gamma(\theta, r^*)},
$$

(31)

the first-order autocorrelation of consumption is

$$
\rho_c \equiv \text{corr}(\Delta c_t, \Delta c_{t-1}) = 0,
$$

(32)

\(^{11}\)For different values of $\sigma$ and $\phi_1$, we have the similar pattern of the equilibrium interest rate as in the benchmark case.
and the relative volatility of financial wealth to income is

\[
\mu_{ay} \equiv \frac{sd(\Delta a_{t+1})}{sd(\Delta y_{t+1})} = \frac{1}{\sqrt{2(1+r^*-\phi_1)}} \sqrt{1 - \phi_1 + \frac{(r^*)^2(1-\theta)(1+\phi_1)}{1-(1+r^*)^2(1-\theta)} + \frac{2r^*(1-\theta)(1-\phi_1^2)}{1-\phi_1(1-\theta)(1+r^*)}}.
\]

(33)

Proof. See Appendix 6.2.

Expression (30) shows that RI has two opposing effects on consumption volatility. The first effect is direct through its presence in the expression of \( \Gamma (\theta, r^*) \), whereas the second effect is through the general equilibrium interest rate \( (r^*) \) and is thus indirect. Using the expression of \( \Gamma (\theta, r^*) \), it is straightforward to show that the direct effect of RI is to increase consumption volatility. The intuition is very simple: the presence of the RI-induced noise dominates the slow adjustment of consumption in determining consumption volatility at the individual level. In contrast, the indirect effect of RI will reduce consumption volatility because it reduces the general equilibrium interest rate. Following Caballero (1991) and Wang (2000), we set \( \rho = 0.015, \alpha = 1.5, \sigma = 0.15, \) and \( \phi_1 = 0.95 \). The second to fifth rows of Table 1 reports how the four key second moments of the joint dynamics of consumption, income, and wealth vary with \( \theta \). It is clear from the second row of Table 1 that the relative volatility of consumption to income is increasing with the degree of inattention. That is, the direct effect of inattention dominates its indirect general equilibrium effect. We can also get the same conclusion by shutting down the general equilibrium (GE) channel, see the corresponding partial equilibrium (PE) results reported in the same table. Comparing the GE and PE results in Table 1, we can see the values of \( \mu_{cy} \) are slightly lower in the GE case. In other words, the general equilibrium effect of RI tends to reduce the volatility of individual consumption.\(^{12}\)

The third row of Table 1 shows that the correlation between consumption and income is increasing with \( \theta \), and the impact of \( \theta \) on this correlation is very significant. For example, when \( \theta \) reduces from 1 to 0.2, \( \rho_{cy} \) reduces from 0.9874 to 0.1534. In addition, we can also see that the general equilibrium effect only has very tiny impact on the correlation between consumption and income.

Expression (32) shows that the first-order autocorrelation of consumption growth is zero. The main reason for this result is that although the slow learning mechanism governed by \( \theta \) leads to higher autocorrelation of consumption growth, the interaction of the noise terms \( (\xi) \) in \( \Delta c_t \) and

\(^{12}\)We cannot examine the stochastic properties of aggregate consumption dynamics because all idiosyncratic shocks (income shocks and RI-induced noise shocks) cancel out after aggregating across consumers.
\[ \Delta c_{t-1} \text{ leads to negative consumption autocorrelation. These two effects are exactly cancelled out and the net effect is zero.}\]

Another important implication of RI in general equilibrium is that RI leads to more skewed wealth inequality measured by \( \mu_{ay} \), the relative volatility of financial wealth to labor income. From the fifth row of Table 1, we can see that when \( \theta \) is reduced from 1 to 0.2, \( \mu_{ay} \) is increased from 2.4783 to 2.7413. From (26), it is clear that the main driving force behind this result is the presence of the estimation error, \( s_t - \hat{s}_t \) because \( \partial \text{var}(s_t - \hat{s}_t) / \partial \theta < 0 \). Note that although \( \partial r^* / \partial \theta > 0 \), the estimation error channel dominates and raises the wealth inequality. Therefore, RI might have the potential to increase the theoretical wealth inequality, which makes the model fit the data better.

3.4. Welfare Losses due to RI in General Equilibrium

We now turn to the welfare cost of RI – how much utility does a consumer lose if the actual consumption path he chooses under RI deviates from the first-best FI-RE path in which \( \theta = 1 \)? To answer this question, we follow Barro (2007) and Luo and Young (2010) to compute the marginal welfare cost due to RI. The following proposition summarizes the main result:

**Proposition 5.** Given the initial value of the state, \( \hat{s}_0 \), the marginal welfare cost (mwc) due to RI is given by

\[
mwc(\theta) \equiv \frac{\partial v(\hat{s}_0)}{\partial \theta} \frac{\theta}{\partial v(\hat{s}_0) / \partial \hat{s}_0} = \frac{\theta}{\partial v(\hat{s}_0) / \partial \hat{s}_0} \left[ (r^* \alpha + \frac{1}{(1 + r^*) \hat{s}_0}) \right] \frac{dr^*}{d\theta},
\]

where \( dr^* / d\theta \) is given in (29) and \( \hat{v}(\hat{s}_0) = -\exp(-r^* \alpha \hat{s}_0 + \ln(1 + r^*)) / (r^* \alpha) \). The monthly dollar loss due to deviating from the FI-RE path (\( \theta = 1 \)) can be written as

\[
\$ \text{loss}(\theta < 1) \equiv \frac{1}{3} r^* mwc(1) (1 - \theta) \hat{s}_0.
\]

**Proof.** See Appendix 6.3. Since we are interested in the deviation from the FI-RE path, \( \theta = 1 \) is considered as the starting point. When the household deviates from 1 to \( \theta \), the percentage change in just \( (\theta - 1) \). \( \hat{s}_0 \) is the given total wealth at the starting point. Finally, we need to convert the change in the \( \hat{s}_0 \) term to monthly rates by multiplying by \( r^* / 3 \).

---

13 Since habit formation leads to the same slow learning mechanism as RI, this result would be consistent with the empirical evidence in Dynan (2000) in which she finds that habit formation does not explain individual consumption behavior.

14 The literature has typically found that simple models based on standard CRRA preferences and on uninsurable shocks to labor income cannot account for the observed U.S. wealth distribution. For example, Aiyagari (1994) finds considerably less wealth concentration in a model with only idiosyncratic labor earnings uncertainty. However, it is worth noting that given the CARA-Gaussian setting, the model here is not suitable to address the issue like why the top 1% or 5% rich families hold a large fraction of financial wealth in the U.S. economy.
Expression (34) gives the proportionate reduction in the initial level of the perceived state \( \hat{s}_0 \) that compensates, at the margin, for a percentage decrease in \( \theta \) (i.e., stronger degree of RI) — in the sense of preserving the same effect on welfare for a given \( \hat{s}_0 \). To do quantitative welfare analysis we need to know the value of \( \hat{s}_0 \). First, we set \( \hat{y}_0 \equiv E[y_t] = 1, \sigma = 0.15, \) and the ratio of the initial level of financial wealth \( (\hat{a}_0) \) to mean income \( (\hat{y}_0) \) equal to 5.\(^{15}\) Second, given that \( \hat{s}_0 = \hat{a}_0 + \hat{y}_0 / (1 + r^* - \phi_1) + \hat{y}_0 / r^* \), we can calculate the values of the monthly dollar loss \( (\$ \text{ loss}) \) for different values of \( \theta \) and the corresponding values of the general equilibrium interest rate. The sixth row of Table 1 reports the welfare losses for different degrees of inattention. For example, when \( \theta = 0.4 \), i.e., when the household deviate from the FI-RE path by 60%, the monthly dollar loss is $7.9, whereas it increases $10.53 when \( \theta = 0.2 \).\(^{16}\)

Another implication of the welfare losses due to RI reported in Table 1 is that there is a general equilibrium effect of RI on the welfare loss. For example, when \( r \) is set to be 1.367% in the PE case, the monthly dollar loss is about 8.4% larger than that obtained in the GE case for any values of \( \theta \).\(^{17}\) Thus, the partial equilibrium results might overstate the welfare losses. These two results thus provide some evidence that it is reasonable for consumers to learn the true state slowly due to finite capacity because the welfare loss from deviating from the FI-RE case is trivial. In other words, although consumers could devote much more capacity to processing economic information and thus improve their consumption decisions, it may be rational for them not to do so even when information costs are negligible.

4. Fixed Information-processing Cost and Elastic Capacity

As argued in Sims (2010), instead of using fixed channel capacity to model finite information-processing ability, one could assume that the marginal cost of information-processing (i.e., the shadow price of information-processing capacity) is fixed. That is, the Lagrange multiplier on (9) is constant. In the univariate case, the objective of the agent with finite capacity in the filtering problem is to minimize the discounted expected mean square error (MSE), \( E \left[ \sum_{t=0}^{\infty} \beta^t (s_t - s^*_t)^2 \right] \),

\(^{15}\)This number varies largely for different individuals, from 2 to 20. 5 is the average wealth/income ratio in the Survey of Consumer Finances 2001. We find that changing the value of this ratio only has minor effects on the welfare implication.

\(^{16}\)The main conclusion here is robust to changes in the values of \( \alpha, \sigma, \) and \( \phi_1 \).

\(^{17}\)Of course, if we set \( r = 0.013799 \) (the value of the equilibrium interest rate when \( \theta = 1 \)), there is no difference between the PE and GE cases because we consider the FI-RE case \( (\theta = 1) \) as the starting point.
subject to the information-processing constraint, or
\[
\min_{\{\Sigma_t\}} \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \Sigma_t + \lambda \ln \left( \frac{R^2 \Sigma_{t-1} + \omega^2_k}{\Sigma_t} \right) \right] \right\},
\]
where \(\Sigma_t\) is the conditional variance at \(t\), \(\lambda\) is the Lagrange multiplier corresponding to (9). Solving this problem yields the optimal steady state conditional variance:
\[
\Sigma = \frac{R^2 (1 - \beta) \tilde{\lambda} - 1 + \sqrt{\left[ R^2 (1 - \beta) \tilde{\lambda} - 1 \right]^2 + 4\tilde{\lambda}R^2}}{2R^2} - \omega^2_k, \tag{36}
\]
where \(\tilde{\lambda} = \lambda / \omega^2_k\) is the normalized shadow price of information-processing capacity. It is straightforward to show that as \(\lambda\) goes to 0, \(\Sigma = 0\); and as \(\lambda\) goes to \(\infty\), \(\Sigma = \infty\). Note that \(\frac{\partial \ln \Sigma}{\partial \ln \omega^2_k} < 1\) if we adopt the assumption that \(\lambda\) is fixed, while \(\frac{\partial \ln \Sigma}{\partial \ln \omega^2_k} = 1\) in the fixed \(\kappa\) case. Comparing (36) with (10), it is clear that the two RI modeling strategies are observationally equivalent in the sense that they lead to the same conditional variance if the following equality holds:
\[
\kappa \left( R, \tilde{\lambda} \right) = \ln R + \frac{1}{2} \ln \left( 1 + \frac{2}{R^2 (1 - \beta) \tilde{\lambda} - 1 + \sqrt{\left[ R^2 (1 - \beta) \tilde{\lambda} - 1 \right]^2 + 4\tilde{\lambda}R^2}} \right). \tag{37}
\]
In this case, the Kalman gain is
\[
\theta \left( R, \tilde{\lambda} \right) = 1 - \frac{1}{R} \left\{ 1 + \frac{2}{R^2 (1 - \beta) \tilde{\lambda} - 1 + \sqrt{\left[ R^2 (1 - \beta) \tilde{\lambda} - 1 \right]^2 + 4\tilde{\lambda}R^2}} \right\}^{-1}. \tag{38}
\]
It is obvious that \(\kappa\) converges to its lower limit \(\kappa = \ln (R) \approx R - 1\) as \(\lambda\) goes to \(\infty\); and it converges to \(\infty\) as \(\lambda\) goes to 0. In other words, using this RI modeling strategy, the consumer is allowed to adjust the optimal level of capacity in such a way that the marginal cost of information-processing for the problem at hand remains constant. Note that this result is consistent with the concept of ‘elastic’ capacity proposed in Kahneman (1973). Given this relationship between \(\lambda\) and \(\theta\) (or \(\kappa\)), in the following analysis we just use the value of \(\theta\) to measure the degree of attention. However, these two RI modeling strategies, inelastic and elastic capacity, have distinct implications for the model’s propagation mechanism if the economy is experiencing regime switching (e.g., before and after the great moderation). With inelastic capacity, the propagation mechanism governed by the
Kalman gain is fixed regardless of changes in fundamental uncertainty, while with elastic capacity the propagation mechanism will change in response to changes in fundamental uncertainty.

The key difference between this elastic capacity case and the fixed capacity case is that $x$ and $\theta$ now depend on both the equilibrium interest rate and labor income uncertainty for a given $\lambda$. The equilibrium interest rate is now determined implicitly by the following function:

$$D \left( \theta \left( r^*, \tilde{\lambda} \right), r^* \right) \equiv \frac{1}{r^*} \left[ \frac{1}{2} (r^* \alpha)^2 \Gamma \left( \theta \left( r^*, \tilde{\lambda} \right), r^* \right) \omega_2^2 - \ln \left( \frac{1 + \rho}{1 + r^*} \right) \right] = 0. \quad (39)$$

Figure ?? illustrates how $r^*$ varies with labor income uncertainty, $\sigma$, for fixed information-processing cost, $\lambda$. It clearly shows that the aggregate saving function is increasing with the interest rate and the general equilibrium interest rate is decreasing with labor income uncertainty.

Table 2 reports how elastic Kalman gain, the general equilibrium interest rate, and the relative volatility of consumption and wealth to income for different values of $\sigma$. We can see from the table that when the economy becomes more volatile (i.e., larger $\sigma$), the Kalman gain ($\theta$) increases while the equilibrium interest rate ($r^*$) decreases. This result is different from that obtained in the fixed capacity case in which $\theta$ and $r^*$ move in the same direction. (See Table 1.) The main reason for this result is that income uncertainty affects the equilibrium interest rate via two channel: (i) the direct channel which leads to higher aggregate savings (the $\omega_2^2$ term in (39)) and (ii) the indirect channel which leads to lower aggregate savings (the $\theta \left( r^*, \tilde{\lambda} \right)$ term in (39)), and the direct channel dominates.

Furthermore, the relative volatility of consumption to income is decreasing with the value of $\sigma$. That is, consumption becomes smoother when income becomes more volatile. This theoretical result can be used to explain the empirical evidence documented in Blundell, Pistaferri, and Preston (2008) that income and consumption inequality diverged over the sampling period they study.\(^{18}\)

5. Concluding Remarks

In this paper we have studied how rational inattention affects the interest rate and the joint dynamics of consumption and income in a Huggett-type general equilibrium model with the CARA-Gaussian specification. Specifically, we explored how RI reduces general equilibrium interest rates via increasing individual precautionary savings and compared the general equilibrium results

\(^{18}\)They create a new panel series of consumption that combines information from PSID and CEX, focusing on the period when some of the largest changes in income inequality occurred. For other explanations for observed consumption and income inequality, see Krueger and Perri (2006) and Attanasio and Pavoni (2012).
with the partial equilibrium results. The key implication of the model is that the general equilibrium effect reduces the welfare cost of imperfect information substantially; given that those costs are already estimated to be small, our work shows that it may easily be optimal for households to operate under substantial uncertainty even if the costs of information are negligible.

6. Appendix

6.1. Deriving the Consumption and Saving Functions in the Huggett-type Model with RI

Given the consumption function (4), the original budget constraint (2) can be rewritten as

\[
a_{t+1} + \phi y_{t+1} + \frac{\phi \phi_0}{r} = (1 + r) a_t + y_t - c_t + \phi (\phi_0 + \phi_1 y_t + w_{t+1}) + \frac{\phi \phi_0}{r}
\]

\[
= (1 + r) \left( a_t + \phi y_t + \frac{\phi \phi_0}{r} \right) - c_t + \zeta_{t+1},
\]

where the \((t + 1)\)-innovation \(\zeta_{t+1} = \phi w_{t+1}\) is Gaussian innovation process with mean zero and variance \(\phi^2 \sigma^2\). Denote \(s_t = a_t + \phi y_t + \phi \phi_0 / r\), the new budget constraint and the consumption function can be rewritten as

\[
s_{t+1} = (1 + r) s_t - c_t + \zeta_t,
\]

\[
c_t = r \left\{ s_t + \frac{1}{r \alpha} \left[ \ln \left( \frac{1 + \rho}{1 + r} \right) - \ln E_t \left[ \exp \left( -r \phi w_{t+1} \right) \right] \right] \right\}.
\]

Under RI, the first-order condition with respective to \(c\) and the Envelope Theorem give us

\[
u'(c_t) = \frac{1}{1 + \rho} E_t \left[ \hat{v}'(\hat{s}_{t+1}) \right],
\]

\[
\hat{v}'(\hat{s}_t) = \frac{1 + r}{1 + \rho} E_t \left[ \hat{v}'(\hat{s}_{t+1}) \right],
\]

which imply that

\[
u'(c_t) = \frac{1}{1 + r} \hat{v}'(\hat{s}_t).
\]

Conjecture that the value function takes the form

\[
\hat{v}(\hat{s}_t) = -\frac{1}{r \alpha} \exp \left[ -r \alpha (\hat{s}_t + b) \right].
\]
Combining the exponential utility, (40) and (41), the candidate optimal consumption is given by

\[ c^*_t = r \left( \hat{s}_t + b + \frac{1}{r\alpha} \ln (1 + r) \right). \]  (42)

Plugging (42) into the utility function gives

\[ u(c^*_t) = -\frac{1}{\alpha} \exp (-\alpha c^*_t) = -\frac{1}{\alpha} \exp \left(-\alpha r \left( \hat{s}_t + b + \frac{1}{r\alpha} \ln (1 + r) \right) \right) = \frac{1}{1 + r} \hat{\nu}(\hat{s}_t). \]  (43)

Substituting (43) into the Bellman Equation (41) leads to

\[ \hat{\nu}(\hat{s}_t) = \frac{1 + r}{1 + \rho} E_t [\hat{\nu}(\hat{s}_{t+1})]. \]  (44)

Using (41) and (12), (44) implies that

\[ c^*_t = r \left\{ \hat{s}_t + \frac{1}{r^2\alpha} \left[ \ln \left( \frac{1+\rho}{1+r} \right) - \ln E_t \left[ \exp \left(-r\alpha \hat{\nu}_{t+1} \right) \right] \right] \right\}. \]

Matching coefficients in (42) and (18) gives

\[ b = -\frac{1}{r\alpha} \ln (1 + r) + \frac{1}{r^2\alpha} \left\{ \ln \left( \frac{1+\rho}{1+r} \right) - \ln E_t \left[ \exp \left(-r\alpha \hat{\nu}_{t+1} \right) \right] \right\}. \]

By utilizing (4), (13) and (18), we can derive the savings function as follows:

\[ d^*_t = ra_t + y_t - c^*_t \]
\[ = ra_t + y_t - c_t + (c_t - c^*_t) \]
\[ = ra_t + y_t - r \left\{ a_t + \phi y_t + \frac{\phi \psi_0}{r} + \frac{1}{r^2\alpha} \left( \ln \left( \frac{1+\rho}{1+r} \right) - \ln E_t \left[ \exp \left(-r\alpha \hat{\nu}_{t+1} \right) \right] \right) \right\} \]
\[ \left\{ r \left[ a_t + \phi y_t + \frac{\phi \psi_0}{r} + \frac{1}{r^2\alpha} \left( \ln \left( \frac{1+\rho}{1+r} \right) - \ln E_t \left[ \exp \left(-r\alpha \hat{\nu}_{t+1} \right) \right] \right) \right] \right\} \]
\[ - r \left[ \hat{s}_t + \frac{1}{r\alpha} \left( \ln \left( \frac{1+\rho}{1+r} \right) - \ln E_t \left[ \exp \left(-r\alpha \hat{\nu}_{t+1} \right) \right] \right) \right] \]
\[ = (1 - \phi_1) \phi (y_t - \bar{y}) + r (s_t - \hat{s}_t) + \frac{1}{r\alpha} \left[ \ln E_t \left[ \exp \left(-r\alpha \hat{\nu}_{t+1} \right) \right] - \ln \left( \frac{1+\rho}{1+r} \right) \right]. \]
6.2. Deriving The Stochastic Properties of the Joint Dynamics of Consumption and Income

Using (27), it is straightforward to compute that

$$\frac{\text{sd} \left( \Delta c^*_{t+1} \right)}{\text{sd} \left( w_{t+1} \right)} = \phi r^* \sqrt{\Gamma \left( \theta, r^* \right)}.$$ 

Combining it with $\text{var} \left( \Delta y_{t+1} \right) = \frac{2}{1 + \theta_1} \text{var} \left( w_{t+1} \right) = \frac{2\sigma^2}{1 + \theta_1}$ yields (30) in the main text.

Using (27) and the expression of $\Delta y_{t+1}$, the covariance between $\Delta c^*_{t+1}$ and $\Delta y_{t+1}$ can be written as:

$$\text{cov} \left( \Delta c^*_{t+1}, \Delta y_{t+1} \right)$$

$$= E \left( \Delta c^*_{t+1} \right) \left( \Delta y_{t+1} \right) - E \left( \Delta c^*_{t+1} \right) E \left( \Delta y_{t+1} \right)$$

$$= E \left[ r^* \hat{s}_{t+1} \right] \left[ w_{t+1} - (1 - \phi_1) \left( \sum_{i=0}^{\infty} \phi_1^i w_{t-i} \right) \right]$$

$$= r^* E \left\{ \left[ \theta R (s_t - \hat{s}_t) + \theta (\xi_{t+1} + \xi_{t+1}) \right] \left[ w_{t+1} - (1 - \phi_1) \sum_{i=0}^{\infty} \phi_1^i w_{t-i} \right] \right\}$$

$$= r^* \left\{ \theta \phi - \theta R (1 - \theta) (1 - \phi_1) \left[ \phi + (1 - \theta) R \phi_1 \phi + (1 - \theta)^2 R^2 \phi_1^2 \phi + \cdots \right] \right\} \sigma^2$$

$$= \frac{1 - (1 - \theta) R}{1 - (1 - \theta) R \phi_1} r^* \theta \phi \sigma^2.$$

Combining this expression with $\text{sd} \left( \Delta c^*_{t+1} \right) = \phi r^* \sqrt{\Gamma \left( \theta, r^* \right)}$, we can obtain the expression for the correlation between $\Delta c^*_{t+1}$ and $\Delta y_{t+1}$:

$$\rho_{cy} = \text{corr} \left( \Delta c^*_{t+1}, \Delta y_{t+1} \right) = \frac{\text{cov} \left( \Delta c^*_{t+1}, \Delta y_{t+1} \right)}{\text{sd} \left( \Delta c^*_{t+1} \right) \text{sd} \left( \Delta y_{t+1} \right)} = \theta \frac{1 - (1 - \theta) R}{1 - (1 - \theta) R \phi_1} \sqrt{\frac{1 + \phi_1}{2 \Gamma \left( \theta, r^* \right)}},$$

which is just (30) in the main text.
To derive the first-order autocorrelation of consumption growth, we first the covariance:

\[
\text{cov}(\Delta c^*_t, \Delta c^*_t+1) = E(\Delta c^*_t \Delta c^*_t+1) - E(\Delta c^*_t) E(\Delta c^*_t+1)
\]

\[
= r^2 E \left[ \theta R (s_t - \hat{s}_t) + \theta (\hat{s}_{t+1} + \hat{s}_{t+1}) \right] \left[ \theta R (s_{t-1} - \hat{s}_{t-1}) + \theta (\hat{s}_t + \hat{s}_t) \right]
\]

\[
= r^2 E \left\{ \left[ \theta R (1 - \theta) \sum_{i=0}^{\infty} (1 - \theta)^i R^i \hat{s}_{t-i} - \theta^2 R \sum_{i=0}^{\infty} (1 - \theta)^i R^i \hat{s}_{t-i+1} \right] \cdot \left[ \theta R (1 - \theta) \sum_{i=0}^{\infty} (1 - \theta)^i R^i \hat{s}_{t-i} - \theta^2 R \sum_{i=0}^{\infty} (1 - \theta)^i R^i \hat{s}_{t-i+1} + \theta (\hat{s}_t + \hat{s}_t) \right] \right\}
\]

\[
= r^2 \left\{ \theta^2 R (1 - \theta) E [\hat{s}_t^2] + \theta^2 R^2 (1 - \theta)^2 \left[ (1 - \theta) R + (1 - \theta)^3 R^3 + \cdots \right] E [\hat{s}_t^2] \right\}
\]

\[
= 0,
\]

which implies that the consumption growth are uncorrelated.

Given \(\Delta a^*_t+1 = \frac{1-\phi_1}{1+r^* - \phi_1} (y_t - \bar{y}) + r^* (s_t - \hat{s}_t)\), we first derive the variance of the current account as follows:

\[
\text{var}(\Delta a^*_t+1) = \text{var}\left( \frac{1-\phi_1}{1+r^* - \phi_1} (y_t - \bar{y}) + r^* (s_t - \hat{s}_t) \right)
\]

\[
= \left( \frac{1-\phi_1}{1+r^* - \phi_1} \right)^2 \sigma^2 + \frac{1-\theta}{1 - (1+r^*)^2 (1-\theta)} (r^* \omega) \right)^2 + 2r^* \left( \frac{1-\phi_1}{1+r^* - \phi_1} \right) \text{cov}(y_t - \bar{y}, s_t - \hat{s}_t)
\]

\[
+ \left( \frac{1-\phi_1}{1+r^* - \phi_1} \right)^2 \sigma^2 + \frac{1}{1 - (1+r^*)^2 (1-\theta)} (1+r^* - \phi_1)^2\]

\[
\text{cov}\left( \frac{\omega_t}{1 - \phi_1 \cdot L} \cdot \frac{(1 - \theta) \hat{s}_t}{1 - (1 - \theta) (1+r^*) \cdot L} \right)
\]

\[
= \left( \frac{1-\phi_1}{1+r^* - \phi_1} \right)^2 \sigma^2 + \frac{(r^*)^2 (1 - \theta)}{1 - (1+r^*)^2 (1-\theta)} (1+r^* - \phi_1)^2\]

\[
+ 2r^* \left( \frac{1-\phi_1}{1+r^* - \phi_1} \right) \sigma^2 + \frac{1}{1 - \phi_1 (1-\theta) (1+r^*)} \cdot \frac{\omega_t}{1 - \phi_1 \cdot L} \cdot \frac{(1 - \theta) \hat{s}_t}{1 - (1 - \theta) (1+r^*) \cdot L} \right)
\]

\[
= \frac{\sigma^2}{(1+r^* - \phi_1)^2}\left[ 1 - \frac{\phi_1}{1+r^* - \phi_1} + \frac{(r^*)^2 (1 - \theta)}{1 - (1+r^*)^2 (1-\theta)} + 2r^* (1 - \theta) \frac{(1-\phi_1)}{1-\phi_1 (1-\theta) (1+r^*)} \right],
\]

where we use the fact that \(\hat{s}_t\) is independent of \(\omega_t\) and \(\hat{s}_t\). Using the expression that \(\text{var}(\Delta y_{t+1}) = 2\sigma^2 / (1 + \phi_1)\), we can obtain (33) in the main text.
6.3. Computing the Welfare Loss due to RI

Given that the value function under RI in general equilibrium is
\[
\hat{v}(\hat{s}_0) = -\frac{1}{r^* \alpha} \exp \left( -r^* \alpha \hat{s}_0 + \ln (1 + r^*) \right),
\] (45)
we can compute the following partial derivatives:
\[
\frac{\partial \hat{v}(\hat{s}_0)}{\partial \theta} = \exp \left( -r^* \alpha \hat{s}_0 \right) \frac{r^*}{r^* + \alpha \hat{s}_0 (1 + r^*)} \frac{dr^*}{d\theta},
\]
\[
\frac{\partial \hat{v}(\hat{s}_0)}{\partial \hat{s}_0} = (1 + r^*) \exp \left( -r^* \alpha \hat{s}_0 \right).
\]
The marginal welfare cost due to RI can thus be written as:
\[
mwc \equiv \left( \frac{\partial v(\hat{s}_0)}{\partial \theta} \right) \theta \left( \frac{\partial v(\hat{s}_0)}{\partial \hat{s}_0} \right) \hat{s}_0 \frac{dr^*}{d\theta},
\]
where we use the facts that in general equilibrium (i.e., \( \ln \left( \frac{1}{1 + r^*} \right) = \ln E_t \left[ \exp \left( -r^* \alpha \hat{z}_{t+1} \right) \right] \)), and \( dr^*/d\theta \) is given in (29).

6.4. An Extension to Incorporate Durable Consumption

Following Bernanke (1985) and Gali (1993), we consider an FI-RE version of the PIH model which includes both durable and nondurable consumption. The optimizing decisions of a representative consumer in the RE-PIH model with durables goods can be formulated as
\[
\max_{\{c_t, k_{t+1}\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, k_t) \right],
\] (46)
subject to the budget constraint
\[
a_{t+1} = (1 + r) a_t + y_t - c_t - e_t,
\] (47)
and the accumulation equation for durables
\[
k_{t+1} = (1 - \delta) k_t + e_t,
\] (48)
where \( u(c_t, k_t) = - \exp (-\alpha_c c_t) / \alpha_c - q \exp (-\alpha_k k_t) / \alpha_k \) is the utility function, \( c_t \) is consumption of nondurables, \( k_t \) is the stock of durables goods, \( e_t \) is the purchase of durable goods, \( \beta = 1 / (1 + \rho) \) is the discount factor, \( R = 1 + r \) is the constant gross interest rate, \( \delta \) is the depreciation rate of durable goods, \( \alpha_c > 0, \alpha_k > 0 \), and \( q > 0 \).

To incorporate RI, following the same procedure used in the our benchmark model and Luo, Nie, and Young (2014), we define a new state variable \((s_t)\) as:

\[
s_t = a_t + \frac{1 - \delta}{R} k_t + \frac{1}{R - \phi} \left( \frac{\phi_0}{R - 1} + y_t \right),
\]

which is governed by the following evolution equation:

\[
s_{t+1} = R s_t - c_t - \frac{R + \delta - 1}{R} k_{t+1} + \tilde{\zeta}_{t+1},
\]

where \( \tilde{\zeta}_{t+1} = \phi_1 w_{t+1} = \frac{1}{R - \phi} w_{t+1} \) is the innovation to \( s_{t+1} \).

Following Luo, Nie, and Young (2014), we formulate the optimization problem for the typical household under RI:

\[
v(\hat{s}_t) = \max_{\{c_t, k_{t+1}\}} E_t [u(c_t, k_t) + \beta v(\hat{s}_{t+1})]
\]

subject to

\[
\hat{s}_{t+1} = R \hat{s}_t - c_t - \frac{R + \delta - 1}{R} k_{t+1} + \tilde{\zeta}_{t+1},
\]

where \( \tilde{\zeta}_{t+1} \) is defined in (14) and \( \hat{s}_0 \) is given. The following proposition summarizes the results from the above dynamic program:

**Proposition 6.** Under RI, the functions of nondurable consumption and the stock of durable accumulation are:

\[
c_t = H_c \hat{s}_t + \Omega_c + \hat{\Pi}_c,
\]

\[
k_{t+1} = \Omega + \frac{\alpha_c}{\alpha_k} c_t,
\]

respectively, where \( H_c = (R - 1) \left( 1 + \frac{R + \delta - 1}{R} \frac{\alpha_c}{\alpha_k} \right)^{-1} \), \( \Omega = -\frac{1}{\alpha_k} \ln \left( \frac{R + \delta - 1}{\alpha_k} \right), \Omega_c = - \left( 1 - \frac{1 - \delta}{R} \right) \left( 1 + \frac{R + \delta - 1}{R} \frac{\alpha_c}{\alpha_k} \right)^{-1} \Omega, \)

\( \hat{\Pi}_c = -\hat{\Pi} / (R - 1), \) and

\[
\hat{\Pi} \equiv \frac{1}{\alpha_c} \ln (\beta R) + \frac{\alpha_c}{2} \left( \frac{R - 1}{R - \phi} \right)^2 \left( 1 + \frac{R + \delta - 1}{R} \frac{\alpha_c}{\alpha_k} \right)^{-2} \Gamma (r, \theta) \phi^2 \sigma^2.
\]
Proof. See Appendix 6.5.

Given the original budget constraint and the decision rules, the expression for individual saving, \( d_t \equiv (R - 1) a_t + y_t - c_t - [k_t + 1 - (1 - \delta) k_t] \), can be written as:

\[
d_t = \frac{R - 1}{R - \phi_1} \left[ 1 - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \left( 1 + \frac{R + \delta - 1}{R} \frac{\alpha_c}{\alpha_k} \right)^{-1} \right] \hat{\zeta}_t + (R - 1) (s_t - \hat{s}_t) \tag{53}
\]

where \( s_t - \hat{s}_t \) is defined in (15), respectively.

After aggregating across all consumers using the same law of large number we applied in the benchmark model, all the idiosyncratic shocks (including the fundamental income shocks and endogenous shocks due to RI) are canceled out and we obtain the following expression for the perceived aggregate saving:

\[
D(\theta, r) \equiv \left[ \frac{R}{H_c} - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \right] \hat{\Pi} + \frac{R - 1}{R - \phi_1} (y_t - \bar{y}), \tag{54}
\]

where \( \hat{\Pi} \) is defined in (52). The following proposition shows the existence of the general equilibrium and the PIH holds in such an equilibrium:

**Proposition 7.** There exists at least one equilibrium with an interest rate \( r^* \in (0, \rho) \) in the RI precautionary-savings model with durables. In any such equilibrium, the aggregate saving is zero:

\[
\frac{1}{2} \left( \alpha_c r^* \right)^2 \left( 1 + \frac{\delta + r^* \alpha_c}{1 + r^* \alpha_k} \right)^{-2} \Gamma(r^*, \theta) \omega_\zeta^2 - \ln \left( 1 + \frac{\rho}{1 + r^*} \right) = 0.
\]

the PIH still holds since consumption follows

\[
c_t = H_c \hat{s}_t + \Omega_c.
\]

**Proof.** The proof is the same as that in the benchmark model. Here we need to use the condition that \( \frac{R}{H_c} - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \neq 0 \).\(^{19}\)

We now examine how rational inattention affects the equilibrium interest rate in the CARA-Gaussian setting with durable consumption. Luo, Nie and Young (2014) show that if \( \frac{\alpha_c}{\alpha_k} = \frac{R + \delta - 1}{e} \) holds, then the model here is observational equivalent to the LQ Gaussian model with durables.

\(^{19}\)Note that the probability measure of \( \frac{R}{H_c} - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) = 0 \) is zero.
Using the estimated parameters in Bernanke (1985) \( (R = 1.01, \delta = 0.025, \text{ and } \varrho = 0.0286) \) and \( \alpha_c = 2 \) used in Caballero (1991) and Wang (2000), we calibrate the CARA parameter on durable consumption: \( \alpha_k = 1.63 \). Given the following parameter values: \( \alpha_c = 2, \alpha_k = 1.63, \sigma = 0.15, \phi_1 = 0.95, \rho = 0.015, \delta = 0.025, \text{ and } \varrho = 0.0286 \), Figure ?? illustrates how \( r^* \) varies with the value of \( \theta \). The figure also clearly shows that the aggregate saving function is increasing with the interest rate and the general equilibrium interest rate is decreasing with the degree of inattention, which is consistent with the conclusion obtained in our benchmark model without durable goods.

6.5. Deriving the Optimal Decisions of the CARA model with Durable Goods

In the FI-RE case, combining the budget constraint (2) and the accumulation equation for durables (48), we can formulate the Lagrange function as follows:

\[
L = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{\alpha_c} \exp\left(-\alpha_c c_t\right) - \frac{\theta}{\alpha_k} \exp\left(-\alpha_k k_t\right) - \lambda_t \left[ a_{t+1} - (Ra_t + (1 - \delta_k) k_t - k_{t+1} + y_t - c_t) \right] \right\} \right],
\]

where \( \lambda_t \) is the Lagrange multiplier. The first-order conditions w.r.t \( c_t, k_{t+1}, \text{ and } a_{t+1} \) are:

\[
\begin{align*}
\lambda_t &= \exp\left(-\alpha_c c_t\right), \\
\lambda_t &= \beta E_t \left[ \varrho \exp\left(-\alpha_k k_{t+1}\right) + (1 - \delta_k) \lambda_{t+1} \right], \\
\lambda_t &= \beta RE_t \left[ \lambda_{t+1} \right],
\end{align*}
\]

respectively. Combining the above three FOCs yields:

\[
E_t \left[ \exp\left(-\alpha_k k_{t+1}\right) \right] = \beta RE_t \left[ \exp\left(-\alpha_k k_{t+2}\right) \right], \tag{55}
\]

\[
k_{t+1} = \frac{\alpha_c}{\alpha_k} c_t + \Omega, \tag{56}
\]

where \( \Omega = -\frac{1}{\alpha_k} \ln \left( \frac{1 - (1 - \delta) / R}{\beta \varrho} \right) \).

Guess that

\[
c_{t+1+j} = c_{t+j} + \Pi_{t+j} + v_{t+1+j}, \tag{57}
\]

where \( \Pi_{t+j} \) is a deterministic term and \( v_{t+1+j} \) is the innovation in consumption. Substituting (56) into (57) yields:

\[
k_{t+j+1} = k_{t+j} + \tilde{\Pi}_{t+j-1} + \tilde{v}_{t+j}, \tag{58}
\]
where $\Pi_{t-1+j} \equiv \frac{a_k}{a_c} \Pi_{t-1+j}$ and $\tilde{v}_{t+j} \equiv \frac{a_c}{a_k} v_{t+j}$. Iterative substitutions in equations (57) and (58) yield:

$$c_{t+i} = c_t + \sum_{j=1}^{i} \Pi_{t+j-1} + \sum_{j=1}^{i} v_{t+j}, \quad i \geq 1, \quad (59)$$

$$k_{t+i+1} = k_{t+1} + \sum_{j=1}^{i} \tilde{\Pi}_{t+j-1} + \sum_{j=1}^{i} \tilde{v}_{t+j}, \quad i \geq 1. \quad (60)$$

Taking conditional expectations, $\sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i E_t [\cdot]$, on both sides of (59) and (60) give:

$$\sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i E_t [c_{t+i}] = \frac{R}{R-1} c_t + \sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=1}^{i} \Pi_{t+j-1}, \quad (61)$$

$$\sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i E_t [k_{t+i+1}] = \frac{R}{R-1} k_{t+1} + \sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=1}^{i} \tilde{\Pi}_{t+j-1}. \quad (62)$$

where $\sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=1}^{i} E_t [v_{t+j}] = \sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=1}^{i} E_t [\tilde{v}_{t+j}] = 0$. Substituting (55) into (58), we have

$$\Pi_t = \left( \frac{a_k}{a_c} \right) \tilde{\Pi}_t = \frac{1}{a_c} \ln (\beta R) + \frac{1}{a_c} \ln (E_t [\exp (-a_c v_{t+1})]). \quad (63)$$

In the next step, iterative substitutions of the combined flow budget constraint lead to the lifetime budget constraint,

$$\sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i c_{t+i} + \sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i [k_{t+1+i} - (1-\delta) k_{t+i}] = R a_t + \sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i y_{t+i}, \quad (64)$$

where

$$\sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i [k_{t+1+i} - (1-\delta) k_{t+i}] = - (1-\delta) k_t + \left( 1 - \frac{1-\delta}{R} \right) \sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i k_{t+1+i}. \quad (65)$$

Taking conditional expectations on both sides of (64) and substituting (56), (61), (62), and (65) into it yield the following consumption functions:

$$c_t = H_t s_t + \Omega \epsilon + \Pi_{t}, \quad (66)$$

$$k_{t+1} = \Omega + \frac{a_c}{a_k} (H_t s_t + \Omega + \Pi_{t}), \quad (67)$$
where

\[
H_c \equiv (R - 1) \left[ 1 + \left( 1 - \frac{1 - \delta}{R} \right) \frac{a_c}{a_k} \right]^{-1},
\]
\[
\Omega_c \equiv -\left( 1 - \frac{1 - \delta}{R} \right) \left[ 1 + \left( 1 - \frac{1 - \delta}{R} \right) \frac{a_c}{a_k} \right]^{-1} \Omega,
\]
\[
\Pi_c \equiv -\frac{1}{R - 1} \Pi,
\]
\[
\Pi \equiv \frac{(R - 1)^2}{R} \sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=1}^{i} \Pi_{i+j-1},
\]
\[
s_t \equiv a_t + \frac{1}{R - \phi_1} \left( y_t + \frac{\phi_0}{R - 1} \right) + \frac{1 - \delta}{R} k_t.
\]

In the final step, we need to pin down the deterministic term, \( \Pi \). Substituting (66), (56), (59), and (65) into the lifetime budget constraint (64), we have

\[
Ra_t = \sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i c_{t+i} + \sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i \left[ k_{t+1+i} - (1 - \delta) k_{t+i} \right] - \sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i y_{t+i}
= -\frac{R}{R - \phi_1} \left( \frac{\phi_0}{R - 1} + y_t \right) - \sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=0}^{i-1} \phi_j^i w_{t+i-j} - (1 - \delta) k_t + \frac{R + \delta - 1}{R - 1} \Omega + \left[ 1 + \frac{R + \delta - 1}{R} \frac{a_c}{a_k} \right] \left\{ \frac{R}{R - 1} \left[ H_c \left( a_t + \frac{1}{R - \phi_1} \left( \frac{\phi_0}{R - 1} + y_t \right) + \frac{1 - \delta}{R} k_t \right) + \Omega_c + \Pi_c \right]
+ \sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=1}^{i} \Pi_{i+j-1} + \sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=1}^{i} v_{t+j} \right\},
\]

where we use the fact \( \sum_{i=0}^{\infty} \left( \frac{1}{R} \right)^i \left[ y_{t+i} \right] = \frac{R}{R - \phi_1} \left( \frac{\phi_0}{R - 1} + y_t \right) + \sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=0}^{i-1} \phi_j^i w_{t+i-j} \). This equality implies that

\[
\sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \sum_{j=1}^{i} \phi_j^i w_{t+j} - \left[ 1 + \frac{R + \delta - 1}{R} \frac{a_c}{a_k} \right] v_{t+j} = 0.
\]

This is satisfied for all \( t \) if and only if the following condition holds:

\[
\sum_{i=1}^{\infty} \left( \frac{1}{R} \right)^i \left[ \phi_j^i w_{t+i} - \left( 1 + \frac{R + \delta - 1}{R} \frac{a_c}{a_k} \right) v_{t+i} \right], \text{for all } h.
\]

Equation (68) implies that:

\[
v_t = \frac{R - 1}{R - \phi_1} \left( 1 + \frac{R + \delta - 1}{R} \frac{a_c}{a_k} \right)^{-1} w_t, \text{ for all } t.
\]
From (63) and (69), we have

$$\Pi = \frac{1}{\alpha_c} \ln (\beta R) + \frac{\alpha_c}{2} \left( \frac{R - 1}{R - \phi_1} \right)^2 \left( 1 + \frac{R + \delta - 1}{R} \frac{\alpha_c}{\alpha_k} \right)^{-2} \sigma^2.$$  

To derive the individual savings function, we construct the new state variable $\tilde{s}_t$ from the consumption function

$$c_t = H_c \tilde{s}_t,$$  \hspace{1cm} (70)

where $\tilde{s}_t = s_t + M$ and $M = \frac{\Omega_c}{H_c} + \frac{\Pi_c}{H_c}$. Using the flow budget constraint, we can derive the transition equation of the new state variable

$$\tilde{s}_{t+1} = R \tilde{s}_t + y_t - c_t - [k_{t+1} - (1 - \delta) k_t] + \frac{1}{R - \phi_1} \left( \frac{\phi_0}{R - 1} + y_{t+1} \right)$$

$$+ \frac{1 - \delta}{R} k_{t+1} - R \left[ \frac{1}{R - \phi_1} \left( \frac{\phi_0}{R - 1} + y_t \right) + \frac{1 - \delta}{R} k_t \right] + (1 - R) M$$

$$= R \tilde{s}_t - c_t - \frac{R + \delta - 1}{R} k_{t+1} + \frac{1}{R - \phi_1} \omega_{t+1} + (1 - R) M$$

$$= R \tilde{s}_t - \left[ 1 + \frac{R + \delta - 1}{R} \frac{\alpha_c}{\alpha_k} \right] c_t - \frac{R + \delta - 1}{R} \Omega + \frac{1}{R - \phi_1} \omega_{t+1} + (1 - R) M$$

$$= \tilde{s}_t + \frac{1}{R - \phi_1} \omega_{t+1} - (R - 1) \frac{\Pi_c}{H_c},$$

which means that the optimal evolution of the new state $\tilde{s}_t$ follows a random walk.

Finally, we can obtain the expression for saving as follows:

$$d_t = (R - 1) a_t + y_t - c_t - (k_{t+1} - (1 - \delta) k_t)$$

$$= (R - 1) s_t - c_t - \left( \Omega + \frac{\alpha_c}{\alpha_k} c_t \right) + \frac{1 - \delta}{R} \left( \Omega + \frac{\alpha_c}{\alpha_k} c_{t-1} \right) + \frac{1 - \phi_1}{R - \phi_1} (y_t - \bar{y})$$

$$= (R - 1) \tilde{s}_t - (R - 1) M - \left[ 1 + \frac{\alpha_c}{\alpha_k} \right] (c_{t-1} + \Pi + v_t) - \frac{R + \delta - 1}{R} \Omega + \frac{1 - \delta}{R} \frac{\alpha_c}{\alpha_k} c_{t-1} + \frac{1 - \phi_1}{R - \phi_1} (y_t - \bar{y})$$

$$= (R - 1) \left( \tilde{s}_{t-1} + \frac{1}{R - \phi_1} \omega_t \right) - (R - 1) M - \left( 1 + \frac{R + \delta - 1}{R} \frac{\alpha_c}{\alpha_k} \right) c_{t-1} - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) (\Pi + v_t)$$

$$- \frac{R + \delta - 1}{R} \Omega + \frac{1 - \phi_1}{R - \phi_1} (y_t - \bar{y})$$

$$= \left[ \frac{R - 1}{R - \phi_1} \omega_t - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) v_t \right] - (R - 1)^2 \frac{\Omega_c}{H_c} - (R - 1) \left( \Omega_c \frac{\Omega_c}{H_c} + \frac{\Pi_c}{H_c} \right) - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \Pi$$

$$- \left( 1 - \frac{1 - \delta}{R} \right) \Omega + \frac{1 - \phi_1}{R - \phi_1} (y_t - \bar{y})$$

$$= \frac{R - 1}{R - \phi_1} \left[ 1 - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \left( 1 + \frac{R + \delta - 1}{R} \frac{\alpha_c}{\alpha_k} \right)^{-1} \right] w_t + \left[ \frac{R}{H_c} - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \right] \Omega + \frac{1 - \phi_1}{R - \phi_1} (y_t - \bar{y}) .$$
Similarly, the individual savings function under RI can be derived as follows

\[ d_t^{RI} = \frac{R - 1}{R - \phi_1} \left[ 1 - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \left( 1 + \frac{R + \delta - 1 \alpha_c}{R} \right)^{-1} \right] \tilde{\xi}_t + \left[ \frac{R}{Hc} - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \right] \hat{\Pi} \tag{71} \]

\[ + \frac{1 - \phi_1}{R - \phi_1} (y_t - \bar{y}) + (R - 1) (s_t - \hat{s}_t). \]

Given the time t available information, the conditional mean of (71) can be rewritten as

\[ \tilde{d}_t = \frac{R - 1}{R - \phi_1} \left[ 1 - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \left( 1 + \frac{R + \delta - 1 \alpha_c}{R} \right)^{-1} \right] \tilde{\xi}_t + \left[ \frac{R}{Hc} - \left( 1 + \frac{\alpha_c}{\alpha_k} \right) \right] \hat{\Pi} + \frac{1 - \phi_1}{R - \phi_1} (y_t - \bar{y}). \tag{72} \]

By using the LLN, (72) gives rise to (54).

References


Figure 1. Effects of RI on Aggregate Saving
Figure 2. Effects of RI on the Equilibrium Interest Rate

Figure 3. Effects of Income Volatility on the Interest Rate in GE (Elastic $\kappa$)
Figure 4. Effects of RI on the Equilibrium Interest Rate

Table 1. Implications of RI for interest rates, Consumption, Wealth, and Welfare

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<td>10.53</td>
<td>7.90</td>
<td>5.27</td>
<td>2.63</td>
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<td>( r )</td>
<td>1.367%</td>
<td>1.367%</td>
<td>1.367%</td>
<td>1.367%</td>
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<tr>
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<td>$loss \</td>
<td>11.41</td>
<td>8.56</td>
<td>5.71</td>
<td>2.85</td>
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Table 2. Implications of RI in GE and PE (Elastic κ)

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<tr>
<th></th>
<th>σ</th>
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<th>0.15</th>
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<td>GE</td>
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<tr>
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<td>θ</td>
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<td>0.239</td>
<td>0.360</td>
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<td>1.482%</td>
<td>1.437%</td>
<td>1.3744%</td>
<td>1.3003%</td>
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<td>0.2481</td>
<td>0.2314</td>
<td>0.2183</td>
<td>0.2066</td>
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<td>2.6774</td>
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<tr>
<td>PE</td>
<td>λ</td>
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<tr>
<td></td>
<td>$r$</td>
<td>1.3003%</td>
<td>1.3003%</td>
<td>1.3003%</td>
<td>1.3003%</td>
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<tr>
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<td>θ</td>
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