Bankruptcy and Delinquency in a Model of Unsecured Debt

Kartik Athreya, Juan M. Sanchez, Xuan S. Tam and Eric R. Young*

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Abstract

The two channels of default on unsecured consumer debt are (i) bankruptcy, which legally grants partial or complete removal of unsecured debt under certain circumstances, and (ii) delinquency, informal default via nonpayment. In the United States, both channels are used routinely. This paper introduces a model of unsecured consumer credit in the presence of both bankruptcy and delinquency and presents three new findings: First, with respect to the choice between bankruptcy and delinquency, labor income shocks matter. Specifically, we find delinquency is readily used by borrowers with the worst labor market outcomes, even those with relatively minor levels of debt. In contrast, bankruptcy is used by households with relatively high debts, but whose long-run earnings prospects are high enough to make interest rate penalties from delinquency too large. Second, financial distress is persistent, in the sense that households in poor financial conditions stay in that state for several quarters. Third, in broad terms, bankruptcy and delinquency are “substitutes,” with bankruptcy increasing as delinquency costs rise.

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*Athreya: Federal Reserve Bank of Richmond, 701 East Byrd Street Richmond, VA, 23219; (804) 697-8000; kartik.athreya@rich.frb.org. Sanchez: Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO, 63166-0442; sanchez@stls.frb.org. Tam: City University of Hong Kong; xuanstam@gmail.com. Young: Department of Economics University of Virginia, 242 Monroe Hall Charlottesville, VA, 22904; ey2d@virginia.edu.
1 Introduction

Personal bankruptcy is a legal procedure that removes unsecured debt obligations subject to some costs. More than 1 million U.S. households have filed for bankruptcy in each of the past two decades. Bankruptcy is, however, not the only route for households to delay or lower debt obligations: they can simply stop paying—and payment delayed can become payment denied. Faced with nonpayment, lenders have the legal right to seize resources from delinquent account holders. Most prominently, lenders may garnish wages, subject to court approval. However, lenders’ ability to seek full repayment is limited because households always retains bankruptcy as an option. Thus, lenders set interest rates on delinquent obligations considering its effect on the repayment probability.

Obtaining debt relief via delinquency appears widespread. Ausubel and Dawsey (2004) analyze data from a large U.S. issuer of MasterCard and Visa card accounts, and find that a nontrivial fraction of accounts become delinquent: 8.8 percent of the debtors were delinquent for at least two months. This is similar to aggregate data for 2008-2012 which indicates that approximately 10 percent of unsecured credit balances were classified as delinquent. Additionally, 1.3 percent of borrowers remained delinquent for long enough to obtain “informal bankruptcy” protection by having their debts written off by lenders. This percentage is of the same order of magnitude as the 1.5 percent who filed for bankruptcy protection in the same period.¹ These authors conclude that “an economic model of consumer lending that assumes bankruptcy as the only alternative to repayment misses an essential branch of the tree” (page 4).

Our goal in this paper is to isolate and understand the nature of the relationship between

¹Their study is representative of approximately the top 50 percent of the U.S. population as ranked by credit history.
these two empirically relevant ways households alter repayment relative to *ex ante* agreements: personal bankruptcy and informal delinquency. The primary contribution of this paper is the tractable addition of a delinquency option (the informal skipping of a promised payment) into an otherwise standard consumption-savings model with bankruptcy. From a quantitative perspective, we calibrate the model to better understand the role that the joint presence of delinquency and bankruptcy play in unsecured credit allocation to U.S. households and how these options help households deal with shocks. Since our model features not just bankruptcy but also delinquency, we can also use it to understand the interactions policies induce between them.\(^2\)

Our model offers three main findings. First, with respect to the choice between bankruptcy and delinquency, wage shocks matter. Specifically, delinquency is readily used by borrowers with the worst labor market outcomes, especially when the latter are persistent, even those with relatively minor levels of debt. In contrast, bankruptcy is used by households with somewhat higher longer-run earnings and higher debt levels, making delinquency less optimal. This occurs because poor future income prospects are necessary for delinquency to yield debt forgiveness, while bankruptcy instead offers a “fresh start” by removing unsecured debts for those able to earn more than the lowest levels of income.

Second, households in financial distress have elevated odds of being in financial distress again soon after. Earnings-related outcomes that make both delinquency and bankruptcy optimal at a given time tend to be persistent and elevate the odds of the recurrence of these events in the near future. This persistence occurs despite the debt forgiveness that bankruptcy, and sometimes delinquency, provides.

\(^2\)This is particularly relevant for understanding novel policy changes, such as those the newly-created Consumer Financial Protection Bureau, of policies regulating debt collection practices for delinquent borrowers. See “CFPB to Supervise Large Debt Collection Firms,” American Banker, October 24, 2012.
Third, in broad terms, bankruptcy and delinquency are “substitutes,” with increases in the costs of one option substantially increasing the use of the other. Our model implies that although high permissible levels of wage garnishment can lower delinquency rates, they may increase bankruptcy rates. It is important to note that this substitution need not occur in equilibrium, as debt levels will also respond to changes in the costs of default; for example, garnishment could reduce borrowing sufficiently to cause a decline in bankruptcy, rather than a rise, even though the risk of bankruptcy would increase for any given debt level.

Our quantitative analysis is based on the model of household-level labor market outcomes of Low, Meghir, and Pistaferri (2010). We chose this model because it offers a rich characterization of risk, especially wage and employment risk. It also allows for workers to reject wage offers, which is important given that in our model debt relief options might a priori affect acceptance of work opportunities. For our purposes, a central aspect of using the Low, Meghir, and Pistaferri (2010) process is that it allows us to parameterize our model to quarterly measures of risk and credit use. This short period is necessary given that delinquency is often short-term in the data (our preferred measure of delinquency involves debt at least 90 days or one quarter past due).

The ability to default on debt without declaration of personal bankruptcy is important for at least two related reasons. First, as mentioned above, the option to informally default through delinquency may provide an alternative to bankruptcy and vice versa. Agents who opt to become or remain delinquent and put off bankruptcy do so because they view it as less costly than bankruptcy, even when the latter leaves any future income “free and clear” (in the case of the predominant form of bankruptcy, Chapter 7), but imposes significant costs on the borrower related to both legal expenses and other costs. The choice of delinquency or bankruptcy depends on the relative short-term costs of the options and on the path of
expected future income. In the case of delinquency, the costs arise from the expense of rolling
over the debt, and the expected levels of wage garnishment.

Importantly, one possibility is that households routinely find delinquency preferable to
bankruptcy, at least initially. That is, delinquency may simply be a stop en route to a
bankruptcy that was always part of a household’s optimal plan. However, it may also be
a gamble by households that, by surviving temporarily via delinquency, they will receive
income draws high enough to avoid any costs associated with bankruptcy. Of course, the
costs of delinquency at any date will rise as expected future household income rises—since
the expected present value of payment via garnishment will rise, all else equal, as well as
any penalty imposed by lenders on borrowers who have skipped payment. Thus, bankruptcy
may even be used by some agents with high expected future incomes, if they have sufficiently
large debts. *Ex ante*, this constellation of outcomes is unlikely, but this possibility shows that
the interaction of these options may be important for the understanding of dates, financial
states, and extent to which households repudiate debt.

Recent work suggests that the relative costs of default matter in a related manner.
Ashcraft, Dick, and Morgan (2007), and later Li, White, and Zhu (2010) and Lilienfeld-Toal
and Mookherjee (2010), all suggest that the Bankruptcy Abuse Prevention and Consumer
Protection Act (BAPCPA), which increased the cost of bankruptcy, led to greater *mortgage*
default, as households worked harder than they otherwise might have to repay unsecured
debts. While our focus is not on the choice between mortgage default and unsecured debt
default (in part because such a focus would require a model of house price declines because
mortgage debt is secured), what is relevant is that there may indeed be a tradeoff between
delayed repayment or nonrepayment in one form versus another.

Our objective in this paper is to model delinquency as an additional option, beyond
formal bankruptcy. Our model of bankruptcy, in turn, most closely resembles the transaction involved in US “Chapter 7” bankruptcy. There is, however, another form of bankruptcy that is widely available to US households: “Chapter 13” bankruptcy. This is a form of bankruptcy that is useful to households who simultaneously own substantial assets and have unsecured debts. Under a Chapter 13 filing, a household proposes a repayment plan to lenders for their unsecured debts, all the while retaining assets that they might hold. While ideally, it would be valuable to incorporate this option as well, it is well beyond the scope of the current paper. Li and Sarte (2006) provide the first, and to date the only, structural model of Chapter 7 and 13 bankruptcy. Our restriction of bankruptcy to its Chapter 7 form is in part technical, but primarily substantive. The technical consideration is that a meaningful model of chapter 13 would require a multiple asset model, as opposed to the single asset model we analyze here, since that is, after all the objective: preserve the use of assets while repudiating debt. More substantively, Chapter 13 is fundamentally different in at least three germane ways. First, in Chapter 13, it is the borrower who proposes a repayment plan, and one that in practice, lenders have little control over (according to Li, 2007, creditors do not typically vote on the repayment plan). Second, delinquent lenders, in contrast, plausibly face a change in borrowing terms, and one that serves lenders well ex-post, subject to the option of the household to default later on. Lastly, the repayment plan of Chapter 13 effectively imposes a tax on labor income (something Li and Sarte, 2006, also allow for). In delinquency, this is not what occurs—any resetting of credit terms does not change the return to working in the current period in the same way: the household can delay payment again, for example. On balance, therefore, our model, especially in its asset structure and simple formulation of the revision of lending terms, allows us to tractably evaluate delinquency. Nonetheless, we recognize that in future work allowing for all three options will be particularly valuable.
Our work is related to recent work of Chatterjee (2010), Chatterjee and Gordon (2012) and Benjamin and Mateos-Planas (2012), though ours employs a life-cycle model to generate a portion of debt from purely intertemporal smoothing motives, as opposed to risk alone. The work of Chatterjee (2010) is a theoretical analysis of the lenders decision to go after delinquent borrowers who have heterogeneous and unobservable costs of filing bankruptcy. We assume the cost of filing bankruptcy is the same for all households, take the lenders collecting efforts as given, and focus on the quantitative implication of allowing for informal default. In particular, and in contrast to Chatterjee and Gordon (2012), we model the process of loan modification explicitly by endogenizing the interest rate imposed on delinquent debt. Our mechanism to reset the interest rate on delinquent debt is simpler than that in Benjamin and Mateos-Planas (2012). They consider a detailed renegotiation process that resembles the reorganization in Chapter 13 bankruptcy. A key feature in their model, which is useful to study reorganization, is that households hold unsecured debts at the same time that they hold assets. As described above, we do not model Chapter 13 bankruptcy but instead focus on informal default.

Regarding the way lenders set interest rates for delinquent borrowers, our model most closely follows that of Kovrijnykh and Szentes (2007). Specifically, upon delinquency, the incumbent lender restates the value of the principal owed to maximize the expected present value of the loan conditional on the borrower’s current state. Our approach imposes that borrowing terms satisfy only what is ex post optimal for lenders, taking as given the household’s outside options in the eventuality of delinquency.

Our work is also related to recent work by Herkenhoff and Ohanian (2012), who study the effect of mortgage default and modifications in the weak recovery of U.S. labor markets after the Great Recession. In their model, mortgages are perpetuities with fixed payments
and a fraction of agents are endowed with a mortgage upon entering the world. Delinquency and loan modifications provide a way to delay payments.\textsuperscript{3}

The paper continues as follows. Section 2 develops the model of debt, delinquency, and bankruptcy. Section 3 characterizes the model. Section 4 describes the parameterization of the model. Section 5 presents our results, focusing on households’ choice between default and bankruptcy. Section 6 concludes.

\textbf{2 Model}

We introduce delinquency into an otherwise standard quantitative consumption-savings lifecycle model with bankruptcy (e.g. Livshits, MacGee, and Tertilt, 2008), where delinquency is a means of delaying repayment, and bankruptcy is a means of repudiating debt once and for all. Both carry costs, with bankruptcy imposing financial costs and costs to utility, while delinquency exposes the borrower to potentially higher future debt obligations and costs to utility as well. We then use the model to address a variety of facts about unsecured credit (e.g., the fraction of borrowers and debt-income ratios), bankruptcy (e.g., filing rates and debt discharged), delinquency (e.g., delinquency rates and households’ delinquency history), and the time path of financial distress.

The central difference between bankruptcy and delinquency is that after delinquency, the borrower still faces debt, a face value that has been reset by the lender who has experienced nonpayment. By contrast, after bankruptcy (Chapter 7 liquidation bankruptcy in particular), the household receives a “fresh start” and owes nothing. Thus, we endogenize the interest rate on delinquent accounts by allowing lenders to “mark up” or “charge off” delinquent accounts. As above, however, their ability to do so (and interest in doing so) is

\textsuperscript{3}They find that foreclosure delays have a significant effect on the unemployment rate.
limited by both the option to declare bankruptcy and the presence of potential lenders that offer new loans at competitive rates.

A key aspect of our model is that it captures the difference between bankruptcy and nonrepayment, and models delinquency in a tractable way. Our model recognizes that competition among lenders forces a creditor to let bygones be bygones, ensuring that the reset face value of debt maximizes the expected payoff to the lender. We locate this debt level and show that it is a value that depends only on the current income of the household and is independent of the initial debt amount.

2.1 Preferences and Endowments

All households are finitely lived, with the head of the household living at most for $J$ periods, and have standard time-separable preferences over consumption. Households vary in their size over the lifecycle, with a household of age $j$ having an effective family size (in adult-equivalent terms) denoted by $\eta_j$. Households receive stochastic endowments as a function of their permanent “type,” $e$, the household’s educational attainment level.

Households have a process for labor income, $y$, that is random and has both age and education-specific components. We will be more specific about this process in a later section, but to conserve on notational burden we specify only that income is Markovian with transition function $\pi(y'|y)$. Finally, households discount the future exponentially, with parameter $\beta$.

At any age $j$, a household wishing to borrow may issue one-period debt with a face value $b_j$ due next period. Households issue all debt to a single lender. In the following period, the household can do one of three things: (i) repay their debts as promised, (ii) file for bankruptcy protection that immediately relieves them of all obligations to repay their debts,
or, as will be emphasized here, (iii) simply not repay their debts as promised, herein referred to as delinquency.

The ability to avoid full repayment when it is due in the following period implies that the household’s debt issued the current period, $b_j$, will be discounted relative to its face value. For a household of age $j$ and education level $e$, a promise to repay $b$ units one period from now is discounted according the price function $q_{j,e}(b_j, y) : B \times Y \rightarrow [0, 1)$. We now detail the effects of each of the three options listed above on current resources and the dynamic consequences of each choice.

### 2.2 Budget Constraints

A household that repays its debts as promised has a completely standard budget constraint, given as

$$c_j + q_{j,e}(b_j, y)b_j = b_{j-1} + y. \quad (1)$$

A household that declares bankruptcy is relieved of any obligations to repay its debts. However, there is an immediate consequence for current period consumption. The household cannot save or borrow in the current period, which implies $b_j = 0$, and bankruptcy consumes real resources, $\Delta(y)$, reflecting court costs and legal fees.\(^4\) Therefore, the budget constraint is trivial, and forces consumption in the period of a bankruptcy filing to satisfy:

$$c_j = y - \Delta(y), \quad (2)$$

Lastly, a household that skips debt payments, but does not seek bankruptcy protection, is

\(^4\)Because some costs can be waived for cause, $\Delta$ will depend on $y$ in the quantitative model. Also, in our quantitative analysis, we will employ a model with period that is very short (one quarter) making the limitation on savings a mild restriction.
said to be in delinquency. In this case, the effect on the household’s budget constraint is that, as with bankruptcy, it cannot borrow or lend in the current period. However, a household that is delinquent faces potential garnishment of its income. As a result, letting $\psi$ denote the proportion of income garnished, the household is left with current period consumption

$$c_j = (1 - \psi) y.$$  

(3)

2.3 Value Functions

At each age, the problem of a household of age $j$ and education $e$ can be represented recursively with a simple state-vector. Let $b_{-1}$ denote debt due in the current period and recall that $y$ denotes current income. These two items are sufficient to spell out the restrictions faced by a household in the current period. Next, let the indicator $d = \{0, 1, 2\}$ denote complete repayment, delinquency, and bankruptcy, respectively. For a household that chooses to repay debt normally in the current period, the payoff is given by

$$v_{d=0}^{j,e}(b_{-1}, y) = \max_b \left\{ u(c) + \beta \sum_{y'} \pi(y' | y) v_{j+1,e}(b, y') \right\}$$  

subject to (1),

$$v_{d=0}^{j,e}(b_{-1}, y) = \max_b \left\{ u(c) + \beta \sum_{y'} \pi(y' | y) v_{j+1,e}(b, y') \right\}$$  

(4)

where $u$ is a period utility function that satisfies standard assumptions.

The remaining lifetime utility of a household that chooses delinquency obeys the func-
tional equation

\[ v_{j,e}^{d=1} (b_{-1}, y) = u(c) - \psi_D + \beta \sum_{y'} \pi (y'|y) v_{j+1,e} (h_{j+1,e} (b_{-1}, y'), y') \]  

subject to (3).

Notice first that the household not only faces wage garnishment in each period but also a utility cost, \( \psi_D > 0 \), reflecting all additional costs associated with remaining delinquent. The key qualitative difference between this choice and bankruptcy (beyond the removal of debt obligations in the latter) is that in the following period the household faces a revised debt obligation, \( h_{j+1,e} (b_{-1}, y') \), the explicit derivation of which is deferred for the moment.

Lastly, when a household invokes bankruptcy protection, the continuation payoff is given by the solution to

\[ v_{j,e}^{d=2} (y) = u(c) - \psi_B + \beta \sum_{y'} \pi (y'|y) v_{j+1,e} (0, y'), \]

subject to (2).

Here, note that the key advantage of bankruptcy over delinquency is that the household will enter the next period with no debt—as seen in the term \( v_{j+1,e} (0, y') \). However, in the current period, we see that with bankruptcy, as with delinquency, household expenditures generate a lower level of utility than they would otherwise. We allow the transactions costs on consumption expenditures from delinquency and bankruptcy to vary, rather than restricting them to be equal. Indeed, they will not be equal: In our calibration, \( \psi_B > \psi_D \); that is, bankruptcy is costlier than delinquency in terms of the current effect on the utility of consumption.
The non-pecuniary costs $\psi_B$ and $\psi_D$ allow for the parsimonious representation of all costs of outright bankruptcy and informal default, respectively. For households, these latter costs include most obviously, any “psychological” costs of lenders’ collections efforts as well as transactions costs associated with poor credit. Moreover, relative to existing work, we more clearly separate the costs of delinquency and bankruptcy because both options are explicitly modeled as available. While we choose to employ non-pecuniary costs, they can also be viewed as a proportional loss of consumption; the implications of the two models are not significantly different.

Given the options available to a household in a given period, their expected maximal lifetime utility satisfies

$$
v_{j,e}(b_{-1}, y) = \max \left\{ v_{j,e}^{d=0}(b_{-1}, y), v_{j,e}^{d=1}(b_{-1}, y), v_{j,e}^{d=2}(b_{-1}, y) \right\}.
$$

As noted, in addition to the effects on current income or utility, the main distinction between bankruptcy and delinquency is that the latter leaves the household with remaining debt obligations. To describe this, consider a household that stops repayments and stands delinquent. In this case, the lender must decide how to restructure the debt. It will choose to reset the face value of debt $b$ to maximize the value of obligations, taking as given the household’s future options to declare bankruptcy, remain delinquent, or become “current” on the debt. Let the mapping from initial delinquent debt $b_{-1}$ and the revised debt, as a function a household’s characteristics, be given by $h_{j,e}(\cdot)$:

$$
h_{j,e}(b_{-1}, y) = \arg\max_b \{b q_{j,e}(b, y)\}.
$$

This function determines the evolution of the face value of a debt in the case of delinquency.
Note that associated with any revision in the delinquent household’s obligation is an implicit interest rate on delinquent debt. We describe this more explicitly further below.

The problem that leads to the function $h_{j,e}(\cdot)$ can be thought simply as a take it or leave it offer by the lender regarding preferred repayment. However, since the borrower has the option of avoiding delinquency by repaying or filing bankruptcy, lenders are constrained in their ability to extract resources from the borrowers. In addition, as lenders choose the new debt level, they are constrained by the fact that the next period borrowers may file bankruptcy or remain delinquent. The price function $q(\cdot)$ must therefore reflect the market’s expectation of future repayment.\footnote{We are implicitly assuming here that there is no commitment on the part of lenders regarding pricing along the delinquency branch — that is, if we had instead assumed that $h_{j,e}(b-1, y)$ was \textit{ex ante} determined by Nash bargaining as in the sovereign debt literature (e.g. Chatterjee and Eyigungor, 2013), without commitment the lender would renege on any agreement and choose (7) \textit{ex post}.}

### 2.4 Pricing Function

The price function is, of course, central in our analysis. Because the possibility of default does not arise in the case of saving, $b \geq 0$, we have:

$$q_{j,e}(b, y) = \frac{1}{1 + r}, \quad (8)$$

where $r$ is the risk-free rate. However, when households borrow, $b < 0$, the price function becomes more complicated. First, and most trivially, there is a per-unit transactions cost for intermediation $\phi$. Second, and substantively, debt prices must not only reflect default risk in the immediately following period, it must also reflect the possibility of repayment following spells of delinquency. It is useful, therefore, that there is a recursive representation
that applies. This is given by the functional equation:

\[ q_{j,e}(b, y) = \frac{Q}{1 + r + \phi}, \]  

(9)

In this expression, \( Q \) is given as follows:

\[
Q = \sum_{y'} \pi(y'|y) 1(d_{j+1,e}(b, y') = 0) + \\
\sum_{y'} \pi(y'|y) 1(d_{j+1,e}(b, y') = 1) \left[ \psi y' + q_{j+1,e}(h_{j+1,e}(b, y'), y') h_{j+1,e}(b, y') \right].
\]

The last equation is the price function for debt with risk of bankruptcy and risk of delinquency. First, consider the states in which the household chooses full repayment, denoted by \( d = 0 \). In these cases, lenders get one dollar per dollar lent. Next, consider the states in which households choose bankruptcy (\( d = 2 \)). Given that lenders obtain nothing in bankruptcy, no term referring to those states is explicitly included. Notice that in bankruptcy both \( d \neq 0 \) and \( d \neq 1 \). Therefore, the right hand side of the preceding equation collapses to zero in all states next period in which bankruptcy is declared. Finally, and more interestingly, focus on states that lead households to choose delinquency, whereby \( d = 1 \). In these cases, the final term on the RHS is activated. Because lenders can, in principle, garnish the household’s income, we obtain the term \( \psi y' \). Next, we adjust the pricing function to reflect the payoffs from creditors’ decisions to adjust the interest rate or face value of debt for the next period. Recall that lenders optimization leads them to choose the debt owed to maximize the market value of debt, \( q_{j+1,e}(h_{j+1,e}(b, y'), y') h_{j+1,e}(b, y') \), where the choice of \( h \) was described in equation (7). Thus, we have a recursive representation for the evolution of debt and interest.
rates along the path in which in households remain delinquent.\textsuperscript{6}

\section{Theoretical Characterization}

We now present a theoretical characterization of our model. Before that, we add an assumption that will hold in all our quantitative exercises. In particular, we assume that the costs of delinquency in the current period are smaller than the costs of bankruptcy in the current period: $\psi_D < \psi_B$. Because bankruptcy generates complete debt forgiveness while delinquency delivers at best partial forgiveness both may be used in equilibrium.

The first property of our model worth highlighting is that $h$ is independent of previous incurred debt obligations $b_{-1}$. As a consequence, hereafter we write it as $h_{j,e}(y)$.\textsuperscript{7} This observation allow us to prove the following results.

\textbf{Lemma 1} $v_{j,e}(b_{-1},y)$ is weakly increasing in $b_{-1}$.

This result is used in the next lemma, which characterizes the default decision in terms of the current amount of debt, $b_{-1}$.

\textbf{Lemma 2} The following statements are true about the current amount of debt, $b_{-1}$, and the decision among bankruptcy, delinquency, and debt repayment:

1. Suppose a household with state $(b_{-1}, y)$ chooses bankruptcy. Then, a household with state $(\hat{b}_{-1}, y)$ with $\hat{b}_{-1} < b_{-1}$ also would choose bankruptcy.

\textsuperscript{6}The fact that one can write these prices as functional equations was previously noticed in models of sovereign default (Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012) and mortgages (Hatchondo, Martinez, and Sanchez, 2013).

\textsuperscript{7}In the presence of interest rate ceilings, this independence may not hold. We are studying the effects of ceilings in ongoing work.
2. Suppose a household with state \((b_{-1}, y)\) chooses delinquency. Then, a household with state \((\hat{b}_{-1}, y)\) with \(\hat{b}_{-1} < b_{-1}\) also would choose delinquency.

3. Suppose a household with state \((b_{-1}, y)\) chooses debt repayment. Then, a household with state \((\hat{b}_{-1}, y)\) with \(\hat{b}_{-1} > b_{-1}\) also would choose debt repayment.

A key implication of these results is that the choice among bankruptcy and delinquency depends on income and not on the current amount of debt.

To better understand delinquency decisions, it is useful to ask first about the alternative to delinquency of simply “rolling over” existing debt. After all, both decisions allow a borrower to avoid making any payment in the current period. Let \(\hat{b}\) be the smallest amount of debt such that the amount of debt \(b_{-1}\) can be rolled over to the next period at the market interest rate; i.e.,

\[
\hat{b}_{j,e}(b_{-1}, y) = \max \{ b : b_{-1} = q_{j,e}(b, y) b \}.
\]

Notice that there will either be one, or no, values of \(\hat{b}\) that attain this maximum, given the prices that the borrower currently faces. If no such debt level exists, the implication is that this household cannot roll over the debt. When a borrower can roll over the debt (when the values of \(e, j, y, b_{-1}\) are such that \(\hat{b}\) exists), let \(i_{j,e}^S(y, b_{-1}) = 1/q_{j,e}(\hat{b}, y)\) be the interest rate charged to roll over \(b_{-1}\). Similarly, let \(i_{j,e}^{DQ}(y, b_{-1}) = h_{j,e}(y) / b_{-1}\) the implicit interest rate faced by a household with debt \(b_{-1}\) that is “rolling over” its debt through delinquency. The next proposition states the main theoretical result.

**Proposition 1** A delinquent household borrows the amount of debt in delinquency until the next period at an implicit interest rate that is not higher than the corresponding market rate \((i_{j,e}^{DQ}(y, b_{-1}) \leq i_{j,e}^S(y, b_{-1}))\).
The intuition is simple. Households in delinquency are, in effect, forcing creditors to lend them the delinquent amount owed. If the interest rate creditors apply to that debt is higher than the market rate available for a households with those characteristics wanting to borrow that amount (in order to roll over the obligation), then the household would strictly prefer avoiding delinquency.

4 Calibration Results

4.1 Quantitative Model

We will now be more specific about the process for $y$ discussed in the previous section. Specifically, we assume it follows the income process estimated by Low, Meghir, and Pistaferri (2010). We made this choice because it allows for employment risk (employment vs. unemployment) and is estimated at a higher frequency (quarterly). As discussed in the introduction, these two features are key to understanding delinquency.

Households vary in their educational attainment, $e$, that can be high (with measure $\Upsilon$) or low (with measure $1 - \Upsilon$), and their age, $a$, which takes values from 22 to 72 with mandatory retirement at 62. Both education and age affect productivity. There is also a persistent shock to productivity, $n$.

In addition, workers are matched with firms with productivity that depends on a match-specific component, $m$, that changes only when the worker changes firm.\(^8\) New draws of match quality come from a normal distribution with mean 0 and variance $\sigma^2_{m,e}$.

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\(^8\)Note that firms do not differ in their productivity. Rather, workers at any time belong in a particular match that determines (in part) their productivity.
Given these parameters, wages \( w(e, a, n, m) \) are given by

\[
\ln(w_a(e, a, n, m)) = x_a(e) + n_a + m_a, \tag{10}
\]

where \( x_a \) is a deterministic age-income profile and \( n_a \) is a random-walk component,

\[
n_a = n_{a-1} + \zeta_a, \tag{11}
\]

where \( \zeta_a \sim N \left( 0, \sigma_{\zeta_{a}}^2 \right) \).

Households may also suffer shocks that lead to disability. In this case, they receive transfers (specified below). Disposable earnings of a household of age \( a \), productivity \( n \), and firm-worker match-specific component \( m \) who are not currently obtaining disability insurance are given by

\[
y(a, n, m, p) = p(w(e, a, n, m))h(1 - \tau) - F_e, \tag{12}
\]

where \( h \) is the fixed number of hours worked by an employed agent, \( \tau \) is the proportional tax rate that used to finances social programs, and \( F_e \) is a fixed commuting cost. When an offer is available, workers decide whether or not to work, denoted by \( p = 1 \) or \( p = 0 \), respectively.

Labor is subject to search frictions, whereby a job offer arrives with probability \( \lambda e^E \) if the household is employed and \( \lambda e^N \) if the household is unemployed. Thus, education also affects the likelihood of reemployment. If a new work opportunity arises, employed workers decide whether to switch jobs or not. When employed, all worker-firm matches are subject to exogenous separation at rate \( \delta e \). Additionally, employed workers can quit to pursue other employment opportunities or become unemployed. Before, we abused notation for the case of a simple income process and we wrote the period utility as \( u(c) \). In this section, the period
utility function is
\[ U(c, p) = \frac{(c \exp(\varphi_e p))^{1-\gamma}}{1 - \gamma} , \]
where \( \gamma \geq 0 \) is the coefficient of relative risk aversion and \( \varphi_e < 0 \) governs the disutility of supplying labor.

There is a social safety net that partially insures workers against the risk of unemployment and the risk of permanent loss of productivity. The former arises from search frictions, and the latter arises from disability, in which case productivity falls to zero. Households receive unemployment payments (with replacement ratio relative to \( n \) of \( \vartheta \) up to a maximum benefit \( \Xi \)) the first period of unemployment, only if they did not quit and continue to receive work offers (stochastically). By contrast, disability is an absorbing state. Disabled individuals may decide whether or not to apply for disability insurance. Individuals may apply for disability if they are older than 50 years old, unemployed, and didn’t apply for disability the last period; applications are successful with probability \( s \in [0, 1] \) and yield benefits
\[
D_{it} = \begin{cases} 
0.9 \times \overline{w} & \text{if } \overline{w} \leq a_1, \\
0.9 \times \overline{w} + 0.32 \times (0.9 \times (\overline{w} - a_1)) & \text{if } a_1 < \overline{w} \leq a_2, \\
0.9 \times \overline{w} + 0.32 \times (0.9 \times (a_2 - a_1)) + 0.15 \times (\overline{w} - a_2) & \text{if } a_2 < \overline{w} \leq a_3, \\
0.9 \times \overline{w} + 0.32 \times (0.9 \times (a_2 - a_1)) + 0.15 \times (a_3 - a_2) & \text{if } \overline{w} > a_3,
\end{cases}
\]
where \( \overline{w} \) is the persistent component of the previous wage.

Lastly, individuals are eligible to receive food stamps, modeled simply as an increment to income \( \Gamma \) rather than a voucher for a specific consumption good. These transfers are represented by the function \( T_j(y) \).
4.2 Parameters, Targets of Calibration, and Fit

We now study the quantitative properties of the model. We focus throughout on stationary
equilibria in which decisions remain constant functions of the household’s state over time.
To maintain comparability to existing work, wherever possible, the parameters are taken
from previous estimations. The parameters are shown in Table 1. Both the income process
and risk-free rate are, for simplicity, modeled as exogenous. We first describe the parameters
taken directly from the work of Low, Meghir, and Pistaferri (2010). The values are presented
in top panel of Table 1. Next, we turn to the parameters that are standard in the literature.
These are seen in the middle panel of the Table. The choice of risk aversion coefficient $\gamma = 2$
is standard in macroeconomics. The annual risk free rate is $r = 1.5$ percent, also standard.
The parameter $\phi$ is set at 3 percent annually to capture the wedge between the interest
rate for credit and deposits not accounted for by the risk of default, and is in line with
values used in previous work (e.g. Athreya (2008)). Filing bankruptcy is costly. The costs
are taken directly from U.S. GAO data, and will depend on the household’s labor status.\footnote{See, e.g. U.S.-Government-Accountability-Office (2008).}
Finally, our benchmark model does not feature wage garnishment. This is to reflect the
empirical regularity that while statutorily garnishment is allowed, it is both limited by both
U.S. Federal and state law, and not prevalent on consumer debts in practice.\footnote{On the legal side, garnishment is limited by the provisions of Title III of the Consumer Credit Protection Act. Lenders seeking to garnish wages or other forms of income (typically non-existent) must obtain approval from the relevant bankruptcy judge and written consent from debtors as well. Federal law places an upper on garnishment equal to 25% of income less deductions for other obligations, including taxes. As a result, consumer lenders receive very low priority in garnishment, as state and local taxes, social security taxes, alimony, child support and student load debts are all deducted prior to a debtor’s income being eligible for garnishment. Moreover, six U.S. states completely rule out garnishing from the outset. Unsurprisingly, therefore, debt collections for the types of consumer debt of interest to us here, overwhelmingly rely on non-legally binding methods such as phone calls and letters. We capture that with the utility cost of delinquency.}

We calibrate three key parameters to match specific targets: $\beta$, the preference dis-
count factor; $\psi_D$, the non-pecuniary cost of delinquency; and $\psi_B$, the non-pecuniary cost of bankruptcy. These three parameters are disciplined using three targeted moments: the bankruptcy rate, the share of debt in delinquency, and the mean of the ratio of assets to income. The obtained parameters and the model’s fit of targeted moments are shown in Table 2. The incidence of bankruptcy, as measured by the bankruptcy rate, and the incidence of delinquency, as measured by the share of debt that is 90 or more days delinquent, are replicated by the model remarkably well. The ratio of asset to income is also closely reproduced by the model.

The obtained values are shown in the bottom panel of Table 1. We note that our benchmark calibration implies that the households under study are slightly less patient than is typically implied by models that assume complete markets. In models such as ours with incomplete markets stemming from uninsurable risk, discount factors of close to 0.95 are not unusual. For example, Davila, Hong, Krusell, and Rios-Rull (2012) feature discount factors below 0.9, as does the earlier estimation of Cagetti (2003); these papers require a low discount factor in order to mitigate strong precautionary savings motives. In contrast, we focus on households’ use of expensive unsecured credit—in order for households to borrow at observed interest rates they must be quite impatient on average.

5 Results

We now employ the quantitative model described above to help answer two questions. First, what is the relationship between a household’s state and their decisions to use delinquency or bankruptcy? Second, what are the implications of systematic changes in the costs of informal default? For the first question, we focus on the behavior of income, employment,
consumption and debt in the periods before and after each type of default: delinquency and bankruptcy. For the second question, we examine a set of counterfactual regimes in which varying amount of labor earnings may be garnished by a creditor to pay a debt obligation.

5.1 Default over the Life-Cycle

We begin by reporting outcomes from our benchmark model for some salient observations on default that were not directly targeted in the calibration. Tables 3 to 5 show model outcomes under the benchmark parameterization. From Table 3, we see that the model produces overall delinquency rates (the fraction of individuals currently behind on obligations) higher than the available empirical measure, though not far off in absolute terms. To be clear, these numbers are stocks, measuring delinquency rates at a given point in time.\textsuperscript{11} The baseline model also captures the rank-ordering of conditional delinquency rates between households of high and low educational attainment, with the latter contributing more to delinquency. In terms of prices, the model performs well: it produces interest rates paid by households that are quite close to SCF data, as shown in Table 4. Finally, Table 5 shows that the mean age and income of those delinquent, bankrupt, or solvent are approximated by the model reasonably well.\textsuperscript{12} Overall, therefore, the benchmark model is both parsimonious, and performs sensibly using primarily non-calibrated parameters (especially those governing income and employment over the lifecycle) with only a few additional internally calibrated

\textsuperscript{11}The data for this table is computed using the Survey of Consumer Finance (SCF) 2004. The questions in this survey do not identify delinquency perfectly. However, the survey asks the following two questions about delinquency: (1) Now thinking of all the various loan or mortgage payments you made during the last year, were all the payments made the way they were scheduled, or were payments on any of the loans sometimes made later or missed? (2) Were you ever behind in your payments by two months or more? According to our definition, to be in delinquency a households needs to have a positive answer to both questions. Additionally, because these questions include all types of debts, to try to keep only delinquent unsecured debt, we exclude the households with no credit cards and those that report paying their mortgage behind schedule.

\textsuperscript{12}In the SCF, households in bankruptcy are those that answered “yes” to the question “Have you ever filed for bankruptcy?” and answered “a year ago” to the question “When was that?”.
While Table 5 presents averages for age and income among debtors, it is of interest to know the model’s implications for the evolution of debt over the lifecycle. As shown in the right-hand panel of Figure 1, unsecured borrowing is more prevalent among young households, and perhaps more interestingly, among those with high education. The main mechanism behind this pattern is intertemporal smoothing. For both education groups, the future (from the perspective of very young households) will be better, on average, than the present. The overall life-cycle profile of income is increasing and it is steeper for those with high education. Both groups borrow, but those with high education borrow more, as shown both in the amount of debt conditional on borrowing (left-hand panel of Figure 1) but also, and, even more clearly, in the proportion of young households choosing indebtedness. The relatively slow decay of the fraction of indebted households also clarifies that the relatively long maturity of unsecured debt. This is consistent with work of Calem, Gordy, and Mester (2005) who show based on annual data that the serial correlation of unsecured debt is above 0.9.

Given our focus on debt default, how are the borrowing patterns described above related to delinquency and bankruptcy? Figure 2 reports the life-cycle behavior of delinquency (left) and bankruptcy (right). Two points are apparent. First, default occurs more or less contemporaneously with debt. Thus, it is not the case that debts are incurred primarily when young and then mainly defaulted on only at much later dates when lifetime income has been more fully realized. In other words, default is not a tool only reserved for use when lifetime income becomes lower than expected, but rather as a consumption smoothing tool at higher frequencies.

Two additional features are noteworthy. First, bankruptcy is less concentrated at young
ages than is delinquency, with delinquency rates declining somewhat more slowly over the lifecycle. Second, Figure 2 shows that while households with different education levels have similar bankruptcy rates over the lifecycle, this is not true for delinquency rates. Instead, delinquency occurs more frequently and earlier for households with high education. The expected age-income profile of this group is steeper. As a result, these households borrow more when young because they expect high average long-run income. Thus, they more often find themselves in financial distress when hit by bad and persistent income shocks. In this case, delinquency is less costly than bankruptcy because it allows households to obtain temporary debt relief. This point will be developed in more detail below.

Having presented default outcomes for households by age, it is critical to describe the decision making that, along with shocks to earnings, led to those outcomes. Figure 3 describes household choices for a representative young household (age 29) as a function of debt at the beginning of the period (x-axis) and the current-period realization of the persistent shock to wages, $n$ (y-axis).\(^{13}\) We see immediately that persistently low-wage states are necessary, and nearly sufficient, to trigger delinquency. That is, when income is low enough, even those with little debt find delinquency optimal. For these borrowers, the intuition is that delinquency offers a way to smooth consumption without paying the costs associated with bankruptcy, and for some, without forgoing consumption by attempting to roll over some, but not all, debt for one or more periods. In contrast, for those with higher debt levels, delinquency offers a significant level of debt forgiveness, arising from the poor income prospects of the households. The necessity of the debt reduction on the part of lenders can be seen in the figure by noticing that absent such forgiveness, higher levels of debt would simply lead to bankruptcy in more states of nature in the near term.

\(^{13}\)We use a fine grid of shocks to accurately capture changes in default decisions. The shocks $n$ take on 80 values, with probabilities being normally distributed about the 40th value.
Having just alluded to it, we turn now to more precisely describing how, in our model, delinquency changes households’ obligations. That is, when do households experience “penalty” interest rates, and when, instead, do they obtain at least partial debt relief? The possibility of both outcomes is reflected in the function \( h(\cdot) \), which transforms existing debt into an updated, and possibly lower, debt. Figure 4 explains how \( h \) is determined. It plots the amount of resources, \(-bq(b, \cdot)\), delivered today to a household that promises to pay back \( b \) next period, conditional on having a persistent component of productivity, \( n \), that is either low or high. First, recall how \( h \) is determined: It is the value of the promised debt repayment, \( b \), that maximizes the current market value of that obligation, \(-bq(b, \cdot)\). For a household with a high persistent component of productivity, \( n_h \), this market value is the highest point of the dashed green line, and is denoted \( h(n_h) \). Similarly, for households with a low persistent component of productivity, \( n_L \), the value of \( h \) is determined using the function described by the blue solid line, and is denoted \( h(n_l) \). Two features are relevant. First, the amount of debt a delinquent debtor will have next period is increasing in the persistent component of income; i.e., \(-h(n_h) > -h(n_l)\) in Figure 4. As explained next, this generates the pattern that leads households leave delinquency for bankruptcy when income rises.

Figure 4 can now be used to show why a household would choose delinquency. Consider a household with current debt equal to point \( A \) on the vertical axis of the figure. Recall that whether the household finds delinquency attractive or not depends on the current level of the persistent components of income, such as \( n \). It also depends on how easy it is for the household to simply roll over debt and avoid default altogether. Consider first a household with high productivity, \( n_h \). This household can rollover that amount of debt in the credit market by promising to pay exactly \( B \) next period. With this strategy, the household does not need to make any payment this period. If the household chooses delinquency, consumption
today will be exactly the same as that under the rollover strategy because there is no debt payment made this period. However, in the next period, the amount owed will be $h(n_H)$, which is strictly larger than $B$. Second, consider a household with a low persistent component of income, $n_L$. Notice that there is no way that this household can rollover the total amount of debt $A$ at the competitive price offered in the credit market conditional on $n_L$. Indeed, this household could at most obtain the amount $C$ in the market. This amount of debt, $C$, implies that the household must repay $A - C$ this period and will owe exactly the amount of debt $h(n_L)$ in the next. Instead, if the household chose delinquency, it would owe $h(n_L)$ next period, just as it would if it had tried to avoid delinquency. Thus the household can, via delinquency, effectively refinance its whole debt obligation $A$ while leaving itself equally situated in the next period. As a result, current consumption will be higher (by the amount $A - C$), and depending on the costs of delinquency in the current period, this household could indeed find delinquency attractive.

5.2 The Timing of Delinquency and Bankruptcy

We now examine the the persistence of, and interplay between, the two forms of default. Figure 5 plots the proportion of households who are delinquent at a given date $t$ conditional on being delinquent at in a reference quarter, which we denote by date $t = 0$.\textsuperscript{14} As the figure shows, delinquency is a very persistent state, even when compared against borrowers of the same mean age. The probability of being delinquent both before and after being delinquent at $t=0$ remains nearly twice as high as for the overall (mean-age-adjusted) group, and even higher still relative to the overall population.

\textsuperscript{14}Due to its persistence and often early occurrence in the life-cycle, in the case of delinquency, we use the first date of delinquency in a person’s lifetime. For the case of bankruptcy, which is more transient (analyzed below), we use a symmetric window.
From Figure 3, we have already seen that bad realizations of persistent shocks trigger default. To see how poor financial conditions prevail among defaulters, we turn to Figure 6. Two conclusions may be drawn from Figure 6. First, households currently in bankruptcy are more likely to have had past delinquency: With bankruptcy at \( t = 0 \), the likelihood of past delinquency is much higher than the age-adjusted rate. Second, many bankruptcies are not preceded by delinquency. The model implies that between 85 – 90 percent of bankruptcy filers will not have been delinquent in the four quarters prior to bankruptcy. It seems likely that we overstate the fraction of bankruptcies that do not follow a spell of delinquency, but the data needed to check are not available.

Lastly, bankruptcy, through its extreme nature whereby all debts are removed and not simply rescheduled or renegotiated, lowers the incidence of future delinquency in the immediate aftermath of \( t = 0 \). Over the longer run, however, past bankruptcy is associated with future delinquencies in almost identical proportions for both education groups.

To what extent is a delinquent borrower likely to have had a past bankruptcy? As shown in 7, the model suggests that the odds of a delinquent borrower having a past bankruptcy is substantially higher for the two reference groups but still low in absolute terms, at between 2 and 3 percent one year prior to a delinquency.

### 5.3 Income and Employment Dynamics Near Default Events

Having displayed the persistence and comovement-related properties of bankruptcy and delinquency, we now examine the use of default, and its two variants, bankruptcy and delinquency, as tools for household risk management. Specifically, we now study the relationship between default and income, education, employment, and debt.

Given the intuitive connection between income and credit use and default, a natural
starting point is the behavior of income in the neighborhood of a bankruptcy event, as shown in Figures 8 and 9. In these and in any of the following figures, we focus on a given credit event and normalize the date at which the event occurs to \( t = 0 \). The remaining dates trace the circumstances of this subset, over time before and after the event. The flat dashed-green lines in the figures show the mean incomes of households with the same age as the households that default at \( t = 0 \), and the flat dashed-red line is for the entire population.

The Figures show two immediate and natural implications of the model. First, at the time of default, households using either delinquency (Figure 8) or bankruptcy (Figure 9) respectively, have quarterly incomes substantially lower than the mean income.\(^{15}\) As noted above, borrowers are in general younger than the overall population, so this finding is to be expected. However, it remains true even after we condition on age.

Second, incomes of delinquent debtors are substantially higher (approximately 25 percent) than those of bankruptcy filers. This outcome is partially driven by the higher mean age of bankruptcy filers and the upward sloping age-income profile of all households. However, there is a larger gap between the mean income of bankruptcy filers and solvent households with the same mean age than there is between the mean incomes of delinquent households and solvent households with the same mean age.

The noted variation in income arises primarily from changes in employment status. Because our model allows for variation in wages and, perhaps more importantly, variation in employment status, we can examine how employment, specifically, is related to default. Employment is the dominant force behind overall income for most people, especially the young. Figure 10 shows that the employment rate for both education groups drops sharply in the

\(^{15}\)The results for labor earnings (instead of income) are nearly identical and so not given separately here. The similarity occurs because of the overwhelming importance of labor income for most households—especially relatively young households.
period preceding delinquency, with the low-education group suffering more. This finding is consistent with the idea that delinquency helps create “breathing space” for households dealing with the first instance of unemployment, while bankruptcy is for longer spells of unemployment.

In general, bankruptcy filers are financially worse off than those in delinquency and have been so for a long time (more than one year). This is perhaps natural: bankruptcy carries significant costs that are worth paying only when debts are substantial. But for debts to be substantial, incomes and employment rates in the periods preceding default will be low on average, relative to the (age- and education-adjusted) mean. For example, Figure 11 shows that employment rates fall on average for a full year before bankruptcy.

An interesting aspect of the relationship between unemployment and default is that both forms of default are used disproportionately by younger households when income for younger households is substantially lower than the unconditional mean and also lower than that for the age-adjusted mean (Figures 8 and 9). Figures 10 and 11 show that employment rates, too, are systematically lower than for households with the same mean age as the defaulters. This is a key reason for the persistence in delinquency rates shown previously in Figure 5. However, employment rates are routinely higher for the defaulting debtors than they are for the overall population, reflective of the lower reservation wages of the young.

Given that incomes before and after default events are substantially lower for defaulters than for households overall, it is natural to expect that households in default will show relatively high levels of borrowing (Figures 12 and 13). One sharp difference is apparent, though. There is a larger gap between the debt level of households in bankruptcy and households overall than between the debt level of households in delinquency and households overall. For households in bankruptcy, even five quarters prior to bankruptcy, their average debt
is higher—between $200 (low education) and nearly $600 (high education). By contrast, the average debt of households that become delinquent is identical to their solvent counterparts. In this sense, in equilibrium, delinquency will be harder to predict than bankruptcy. Nonetheless, at high frequencies, both forms of default are preceded by large increases of debt, with high-education households increasing their borrowing the most.

Delinquency and bankruptcy are associated with different dynamics after default event as well. As shown in Figures 12 and 13, since (by definition) bankruptcy eliminates unsecured debt altogether, the balance sheets of households that choose bankruptcy are cleared up and debt accumulation begins again. The reason for this was seen earlier in Figures 8 and 9—earnings remains lower than (even the age-adjusted) mean. As a result, agents can expect future income to be higher than current income, and therefore borrow to smooth consumption.

For the same reasons, post-default dynamics are different for households that chose delinquency. Debt increases slightly relative to overall (age-adjusted) mean debt, but not as substantially as for post-bankruptcy borrowers. Unlike after bankruptcy, after delinquency income returns close to its age-adjusted mean within a few quarters. This highlights the interplay between income, age, and debt in differentiating between those who find delinquency useful, and those who find bankruptcy useful.

In what we have presented so far, we have focused on the “intensive” margin of debt: We have presented conditional moments of the distribution of debt given that debt is present on a household’s balance sheet. A natural question is therefore how representative such moments are. Figures 14 and 15 show the proportion of households who are indebted before and after default and suggest that the noted dynamics hold for nearly all households who enter default. Of course, at $t = 0$, all households are indebted. What is more striking, however,
is the very high proportion of households who remain indebted after default. In the case of bankruptcy, conversely, the proportion drops to zero by definition at the beginning of the period following bankruptcy, but rebounds almost immediately thereafter. This later finding is consistent with income of these households being lower than the mean for their demographic group—especially high-education households. Thus, a trigger of default is persistent spells of misfortune that typically (overwhelmingly) lead to/necessitate credit use after default. And in this case, the extensive margin does not seem to differ across bankruptcy and delinquency.

5.4 Policy Towards Delinquency

Up to this point, our focus has been purely positive, aimed only at understanding the implications of the two channels of debt default as defined by current law. However, a policy-related motivation for our work is to understand the feedback effects present between delinquency policy and bankruptcy.

We now concentrate on income garnishment policy. In the benchmark model, garnishment is set to zero to reflect current practices that severely restrict actual garnishment. Table 6 shows the results for selected objects across garnishment regimes. The main implications of garnishment are as follows: First, garnishment has a quantitatively strong effect on overall delinquency. For example, at even a 10 percent garnishment rate, delinquency rates fall to roughly one-fourth of their benchmark value. At the intensive margin, the share of total debt that is delinquent drops very sharply, from 7.78 percent in the benchmark economy, to just 1.57 percent. At higher garnishment rates, both delinquency rates and the share of debt in delinquency stabilize. This finding suggests a population of debtors for whom the option of delinquency is not particularly valuable and another subset for whom it is. Low,
but positive, garnishment rates appear to push these “marginal delinquents” into becoming current with their payments. Their mean debt-to-income ratio falls by a fourth, from 4.03 to 3.19.

Given that delinquency is being restricted as an option, to what extent households use bankruptcy to deal with reductions in income relative to their income expectations? First, bankruptcy rates rise as delinquency becomes more expensive, as does the share of total debt discharged each year via bankruptcy, a finding consistent with the idea that delinquency and bankruptcy are partial substitute tools of debt relief. This is consistent with our earlier results documenting the different roles that bankruptcy and delinquency play. The former continues to be used for more serious income disruptions than the latter. However, it is not the case that there is a wholly offsetting shift from delinquency to bankruptcy.

Our analysis so far is motivated by the fact that the implications of garnishment on overall bankruptcy and delinquency rates are not \textit{a priori} obvious, and depend on the quantitative strengths of preferences, wage risk, and default costs. However, conditional on personal circumstances, particularly debt, increasing the cost of garnishment should have a more unambiguous effect on default risk, since bankruptcy—which wipes out all debt—becomes relatively cheaper. Figure 16 shows that this is the case for all households.

The model suggests that in the United States, there is a set of borrowers who do not value delinquency very highly, as shown by the decline in the proportion of delinquent households employed. At garnishment rates above 10 percent, no delinquent borrower is employed. Thus, the wage tax arising from a nontrivial garnishment regime is an effective deterrent to delinquency. Of course, part of this outcome is driven by the fact that agents can turn down employment opportunities, and may do so if it makes debt relief sufficiently less costly. On balance, this force, while present in principle, is not quantitatively important, as the income
loss from ignoring work opportunities is very costly (as reflected in the overall invariance of unemployment rates to garnishment).

### 5.4.1 Garnishment and Financial Distress Dynamics

How will garnishment likely matter for the dynamics of debt and employment around default events? Figure 17 shows the effect on mean income at the time of delinquency event across garnishment regimes. Intuitively, harsher garnishment lowers the income of households who find delinquency optimal at the time they choose it. This is unsurprising, as garnishment makes delinquency costs higher under high wages. Interestingly, however, the importance of persistent income risk in driving delinquency (already described) plays a more general role as well. As shown in the figure, incomes are lower in periods prior to and following the default as well. The extensive margin of employment features the same pattern. Figure 19 shows employment rates around delinquency events falling systematically with garnishment. This is a useful feature of our model, as it shows that households considering delinquency (i.e., households for whom near-term delinquency is likely) will also reject employment offers more regularly. This finding makes sense because earnings make it costlier to escape from debt, and act as an implicit tax in states of the world where delinquency is useful. In summary, therefore, garnishment can be expected to shift delinquency towards those with more persistent income loss. By contrast, the garnishment regime has little impact on either the incomes or employment paths around bankruptcy, as shown in Figures 18 and 20.

To understand the role of garnishment for debt dynamics around default, we display in Figures 21 and 22 the path of mean debt for those with a default event at date $t = 0$. We have already seen that delinquency rates fall with garnishment; Figure 21 shows that garnishment also has a substantial effect on the size of delinquencies as well—cutting them
nearly in half when comparing the benchmark to the 10 percent garnishment case. As above, though, garnishment primarily reduces delinquency rates and, as seen in Figure 22, has little effect on bankruptcy rates at the intensive margin of indebtedness. The fact that the immediate effects of garnishment fall primarily on delinquency rates, and earnings and employment around delinquency, is natural given the change in costs that these measures impose. Bankruptcy is still affected by garnishment, however, as shown in the figures, as borrowing costs and debt use are affected for all households in all states.

6 Conclusion

In this paper, we explored the two primary forms of default of unsecured consumer debt: delinquency, whereby borrowers do not repay as initially promised, and bankruptcy, whereby debt is completely removed. Each plays a distinct role and the choice of one is often related to the other. In the data, both delinquency and bankruptcy are used frequently \textit{ex post} to alter obligations. The former merely allows a delay in repayment, with no legal implications for a household’s liability, but where creditors may reset households’ debt, and/or seize their labor income. The latter formally eliminates a debt obligation. The restatement of the debt owed from that point on is, in turn, determined under competitive conditions because households continue to hold bankruptcy, refinancing, and repeated delinquency as options.

Our model sheds light on costs and benefits of each and also helps uncover the limits of bankruptcy protection to alter allocations. Roughly, while existing work suggests that strict bankruptcy laws can change credit terms and borrowing substantially (see Athreya, 2008), our work suggests that this conclusion depends on the alternatives available, notably, the alternative to simply remain delinquent. In particular, we show that stricter control of
delinquency, as defined by a relatively high ability to garnish wages, leads to more risk of bankruptcy and lower welfare (on average).

We have attempted, wherever possible, to discipline our quantitative analysis with available data. In particular, we used data to analyze a variety of the model’s implications related to credit market aggregates and household income processes. The model suggested that formal and informal default interact in a rich manner and in ways dependent on household income. In addition, it is also clear that our model offers additional quantitative implications for the dynamics of household default and consumption worthy of further study. However, such work will require better panel data on income, debt and forms of default than what is currently available.

Lastly, we used a simplification of the model to abstract from labor-leisure choices along the “intensive” (hours) margin. Work of Pijoan-Mas (2006) and others has shown that work effort can be a channel by which households mitigate wage fluctuations—in principle including those induced by garnishment. Our approach ties income more closely to episodes where workers lack an opportunity to supply labor, e.g., unemployment. Future work relaxing this requirement including the intensive margin of labor supply may be useful. However, here again, it would be ideal to have data relating labor hours and income to delinquency and bankruptcy. We hope, therefore, that in the future, with the requisite data, research can advance along those lines.
References


7 Appendix

7.1 Proof of Lemma 1

Notice that $v^{d=1}$ and $v^{d=2}$ are independent of $b_{-1}$. Thus, we must show that $v^{d=0} (b_{-1}, y)$ is increasing in $b_{-1}$. This problem can be written as

$$v^{d=0}_{j,e} (b_{-1}, y) = u (b_{-1} + \omega_{j,e} y_j - q_{j,e} (b^*, y) b^*, n_j) + \beta \sum_{y'} \Pr (y'|y) v_{j+1,e} (b^*, y'),$$
where \( b^* \) is the maximizer. Now, we take \( \hat{b}_{-1} > b_{-1} \) and show that \( v_{j,e}^{d=0} (\hat{b}_{-1}, y) > v_{j,e}^{d=0} (b_{-1}, y) \). It is clear that

\[
v_{j,e}^{d=0} (b_{-1}, y) < u \left( \hat{b}_{-1} + \omega_{j,e} y_j - q_{j,e} (b^* , y) b^*, n_j \right) + \beta \sum_{y'} \text{Pr} (y'|y) v_{j,e} (b^*, y')
\]

because \( u \) is increasing, and

\[
u \left( \hat{b}_{-1} + \omega_{j,e} y_j - q_{j,e} (b^* , y) b^*, n_j \right) + \beta \sum_{y'} \text{Pr} (y'|y) v_{j,e} (b^*, y') \leq v_{j,e}^{d=0} (\hat{b}_{-1}, y)
\]

because \( b^* \) is also available for the state \( (\hat{b}_{-1}, y) \) but it may not be the maximizer. □

7.2 Proof of Lemma 2

These results are straightforward because \( v_{j,e}^{d=0} \) is increasing in \( b_{-1} \) and both \( v_{j,e}^{d=1} \) and \( v_{j,e}^{d=2} \) are independent of \( b_{-1} \). □

7.3 Proof of Proposition 1

Assume, by contraction, that a household with state \( j, e, b_{-1}, y \) chooses delinquency and there exist \( \hat{b}_{j,e} (b_{-1}, y) \) which implies \( i_{j,e}^{x} (y, b_{-1}) < i_{j,e}^{DQ} (y, b_{-1}) \). First, notice that the decision of choosing delinquency implies that

\[
v_{j,e}^{d=1} (y, b_{-1}) > v_{j,e}^{d=0} (y, b_{-1}). \tag{13}
\]
Second, notice that the utility of a household rolling over the debt at the market rate is

\[
V_{j,e}(b_{-1}, y) = \left\{ \frac{\eta_j}{1 - \sigma} \left( \frac{y}{\eta_j} \right)^{1-\sigma} + \beta \sum_{y'} \pi(y'|y) v_{j+1,e} \left( \tilde{b}_{j,e}(b_{-1}, y), y' \right) \right\}.
\]  

(14)

Now, notice that since \( i_{j,e}^S(y, b_{-1}) < i_{j,e}^{DQ}(y, b_{-1}) \) we have that \( \tilde{b}_{j,e}(b_{-1}, y) > h_{j,e}(b_{-1}, y) \).

This implies that

\[
V_{j,e}(b_{-1}, y) > v^{d=1}(b_{-1}, y)
\]

(15)

because \( v(b, y) \) is increasing in \( b \). Since rolling over the debt at the market price implies choosing a particular value of debt, we have that

\[
v^{d=0}(b_{-1}, y) \geq V_{j,e}(b_{-1}, y).
\]

(16)

Using (15) we have that

\[
v^{d=0}(b_{-1}, y) > v^{d=1}(b_{-1}, y),
\]

(17)

which contradicts (13).
Table 1: Parameters Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>High Edu.</th>
<th>Low Edu.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education distribution Υ</td>
<td>0.568</td>
<td>0.432</td>
</tr>
<tr>
<td>Std. deviation of permanent shock σζ</td>
<td>0.106</td>
<td>0.095</td>
</tr>
<tr>
<td>Std. deviation of firm-worker shock σm</td>
<td>0.229</td>
<td>0.226</td>
</tr>
<tr>
<td>Job separation rate Δ</td>
<td>0.022</td>
<td>0.039</td>
</tr>
<tr>
<td>Job arrival rate in unemployment λN</td>
<td>0.709</td>
<td>0.657</td>
</tr>
<tr>
<td>Job arrival rate in employment λE</td>
<td>0.623</td>
<td>0.579</td>
</tr>
<tr>
<td>Fixed cost of work F</td>
<td>$1,213</td>
<td>$1,088</td>
</tr>
<tr>
<td>Disutility of working φ</td>
<td>−0.620</td>
<td>−0.550</td>
</tr>
<tr>
<td>Total hours of work h</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>UI replacement ratio θ</td>
<td>0.750</td>
<td>0.750</td>
</tr>
<tr>
<td>UI cap Ξ</td>
<td>$2,384</td>
<td>$2,384</td>
</tr>
<tr>
<td>DI application successful rate s</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>Maximum food stamp Γ</td>
<td>$800</td>
<td>$800</td>
</tr>
<tr>
<td>DI threshold 1 a1</td>
<td>$1,203</td>
<td>$1,203</td>
</tr>
<tr>
<td>DI threshold 2 a2</td>
<td>$7,260</td>
<td>$7,260</td>
</tr>
<tr>
<td>DI threshold 3 a3</td>
<td>$16,638</td>
<td>$16,638</td>
</tr>
<tr>
<td>Risk aversion γ</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>Risk-free interest rate r</td>
<td>0.375%</td>
<td></td>
</tr>
<tr>
<td>Transaction cost φ</td>
<td>0.75%</td>
<td></td>
</tr>
<tr>
<td>BK filing fee for p = 1 $Δ</td>
<td>$1,200</td>
<td></td>
</tr>
<tr>
<td>BK filing fee for p = 0 $Δ</td>
<td>$600</td>
<td></td>
</tr>
<tr>
<td>Discount factor β</td>
<td>0.957</td>
<td></td>
</tr>
<tr>
<td>Non-pecuniary cost BK ψB</td>
<td>1.786</td>
<td></td>
</tr>
<tr>
<td>Non-pecuniary cost DQ ψD</td>
<td>0.104</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Fit of Targeted Statistics

<table>
<thead>
<tr>
<th>statistics</th>
<th>data</th>
<th>model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of debt in 90+ DQ, %</td>
<td>8.9</td>
<td>7.8</td>
</tr>
<tr>
<td>Bankruptcy rate, %</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>Mean (assets/income)</td>
<td>4.07</td>
<td>3.89</td>
</tr>
</tbody>
</table>

Source: “Share of debt in 90+ DQ” obtained from the “Quarterly Report on Household Debt and Credit” of the Federal Reserve Bank of New York. The bankruptcy rate is obtained from the Bankruptcy Institute. “Mean (assets/income)” is obtained from SCF 2004.

Table 3: Incidence of Delinquency in the Model

<table>
<thead>
<tr>
<th>statistics</th>
<th>data, %</th>
<th>model, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delinquency rate, all</td>
<td>2.03</td>
<td>0.97</td>
</tr>
<tr>
<td>Delinquency rate, low education</td>
<td>2.54</td>
<td>1.27</td>
</tr>
<tr>
<td>Delinquency rate, high education</td>
<td>1.13</td>
<td>0.74</td>
</tr>
</tbody>
</table>


Table 4: Mean Interest Rates

<table>
<thead>
<tr>
<th>statistics</th>
<th>data, %</th>
<th>model, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-education households</td>
<td>12.2</td>
<td>9.7</td>
</tr>
<tr>
<td>Low-education households</td>
<td>13.1</td>
<td>10.1</td>
</tr>
<tr>
<td>All households</td>
<td>12.7</td>
<td>9.9</td>
</tr>
</tbody>
</table>

Table 5: Characteristics of Households with Financial Stress

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solvent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>43.6</td>
<td>41.4</td>
</tr>
<tr>
<td>Income</td>
<td>$64,052</td>
<td>$69,240</td>
</tr>
<tr>
<td>Delinquent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>34.7</td>
<td>37.3</td>
</tr>
<tr>
<td>Income</td>
<td>$21,375</td>
<td>$37,086</td>
</tr>
<tr>
<td>Bankrupt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>33.8</td>
<td>40.8</td>
</tr>
<tr>
<td>Income</td>
<td>$21,644</td>
<td>$45,827</td>
</tr>
</tbody>
</table>


Table 6: The Effect of Garnishment on Delinquency and Bankruptcy

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Benchmark</th>
<th>1%</th>
<th>3%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DQ rate, %</td>
<td>0.97</td>
<td>0.57</td>
<td>0.32</td>
<td>0.27</td>
<td>0.25</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>Debt in DQ, %</td>
<td>7.78</td>
<td>4.25</td>
<td>2.06</td>
<td>1.70</td>
<td>1.57</td>
<td>1.60</td>
<td>1.64</td>
<td>1.65</td>
</tr>
<tr>
<td>BK rate, %</td>
<td>0.26</td>
<td>0.25</td>
<td>0.27</td>
<td>0.29</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>Debt in BK, %</td>
<td>2.42</td>
<td>2.49</td>
<td>2.76</td>
<td>2.84</td>
<td>3.01</td>
<td>2.99</td>
<td>2.94</td>
<td>2.96</td>
</tr>
<tr>
<td>DQ and Employed, %</td>
<td>0.74</td>
<td>0.35</td>
<td>0.08</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>BK and Employed, %</td>
<td>0.18</td>
<td>0.19</td>
<td>0.23</td>
<td>0.25</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>Mean (debt/income), %</td>
<td>4.03</td>
<td>3.57</td>
<td>3.28</td>
<td>3.22</td>
<td>3.19</td>
<td>3.18</td>
<td>3.18</td>
<td>3.18</td>
</tr>
<tr>
<td>People in debt, %</td>
<td>19.12</td>
<td>18.40</td>
<td>17.94</td>
<td>17.84</td>
<td>17.65</td>
<td>17.66</td>
<td>17.67</td>
<td>17.67</td>
</tr>
</tbody>
</table>

Table 7: Welfare Gains of Increasing Garnishment

<table>
<thead>
<tr>
<th>Statistics</th>
<th>BM</th>
<th>1%</th>
<th>3%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare gains, CE %</td>
<td>-</td>
<td>-0.16</td>
<td>-0.31</td>
<td>-0.37</td>
<td>-0.37</td>
<td>-0.38</td>
<td>-0.37</td>
<td>-0.37</td>
</tr>
<tr>
<td>Mean c unemployed / Mean c employed</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Variance of log c</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>Mean c young / Mean c old</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Mean (assets/income), %</td>
<td>3.89</td>
<td>3.90</td>
<td>3.90</td>
<td>3.90</td>
<td>3.91</td>
<td>3.91</td>
<td>3.91</td>
<td>3.91</td>
</tr>
</tbody>
</table>

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Figure 1: Life-cycle Profile of Debt by Education Group

Figure 2: Delinquency (Left) and Bankruptcy (Right) over the Lifecycle
Figure 3: Repayment and Productivity

Figure 4: Why Delinquency?
Figure 8: Income around DQ

Figure 9: Income around BK

Figure 10: Employment around DQ

Figure 11: Employment around BK
Figure 12: Debt around DQ

Figure 13: Debt around BK

Figure 14: Share in Debt around DQ

Figure 15: Share in Debt around BK
Figure 16: Interest Rate, Benchmark vs. Garnishment

Figure 17: DQ, Income, and Garnishment

Figure 18: BK, Income, and Garnishment